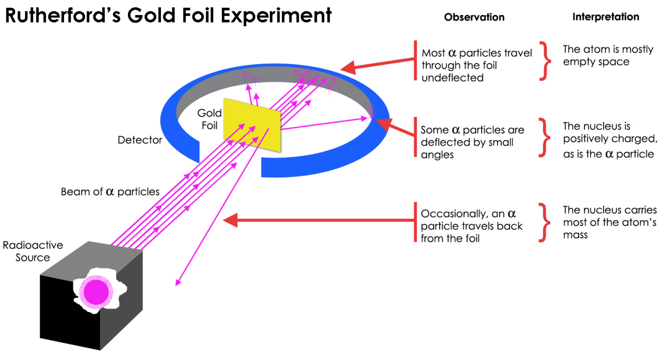
What is...the Yoneda lemma?

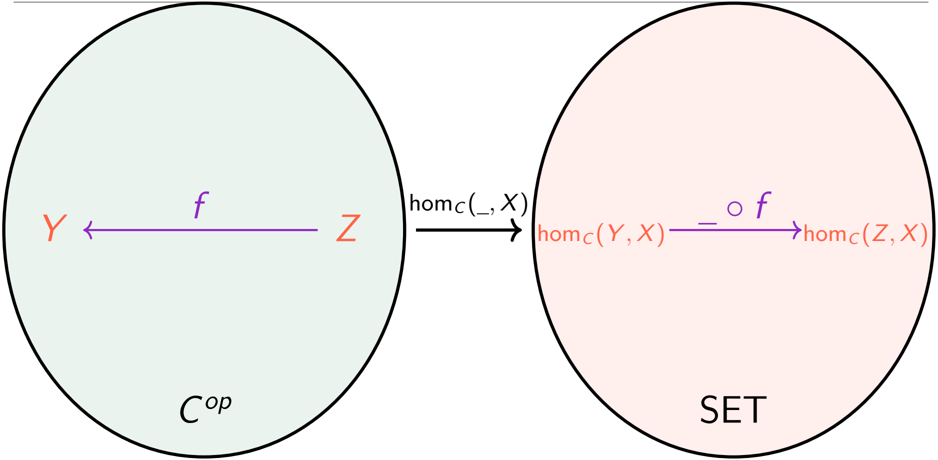
Or: Shoot on sight

Shooting at gold foils



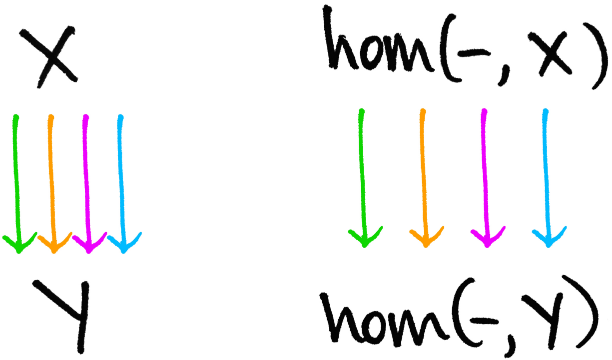
- ∼1910 Geiger–Marsden (Rutherford's gold foil) made landmark experiments
- Idea (reinterpreted) Shoot on X to learn more about X
- The same idea works in mathematics and category theory

The hom functors



- From any C and any $X \in C$ there are always hom functors C^{op} to SET
- They are given by $\hom_C(\_, X)$ on objects
- They are given by precomposition on arrows

Shooting at objects



The Yoneda perspective is the gold foil experiment of category theory, *e.g.*:

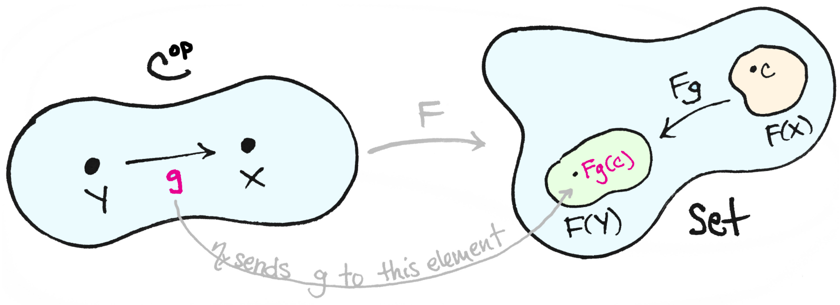
- Everything you want to know about X is encoded in $\hom_C(\_, X)$
- $X \cong Y$ if and only if $\hom_C(\_, X) \cong \hom_C(\_, Y)$ as functors

For completeness: A formal statement

The Yoneda lemma generalizes these observations!

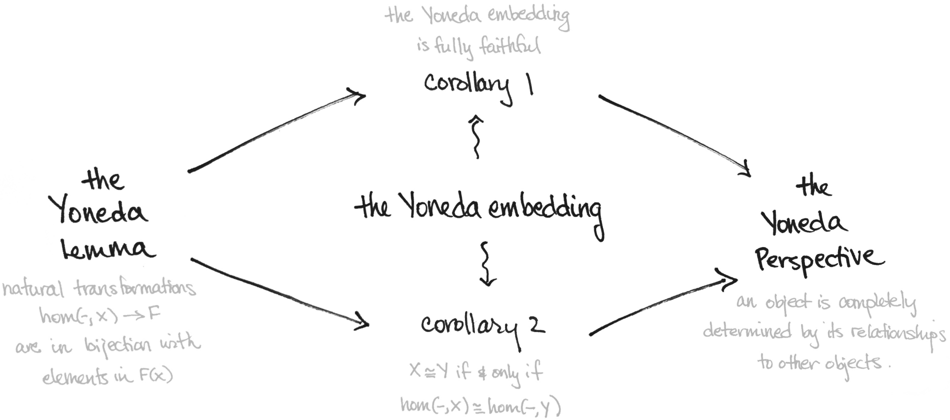
For $F\colon C^{op} \to \mathrm{SET}$ and $X \in C$, nat trafos $\hom_C(\_, X) \Rightarrow F$ are in bijection with $F(X)$

- The set of nat trafos $\hom_C(\_, X) \to F$ could be massive, but Yoneda says its not!
- The nat trafos that exist are those which can be cooked up from $F(X)$
- How does this work? Well:



η_Y is the nat trafo that sends $g\colon Y \to X$ to $F(g)(c)$

Yoneda embedding



- Let $[C, \mathsf{SET}]$ denote the category of functors $C \to \mathsf{SET}$
- **Theorem** There is an embedding $C^{op} \to [C, \mathsf{SET}]$
- This is a(n opposite) gold foil experiment: C is determined by $[C, \mathsf{SET}]$

Thank you for your attention!

I hope that was of some help.