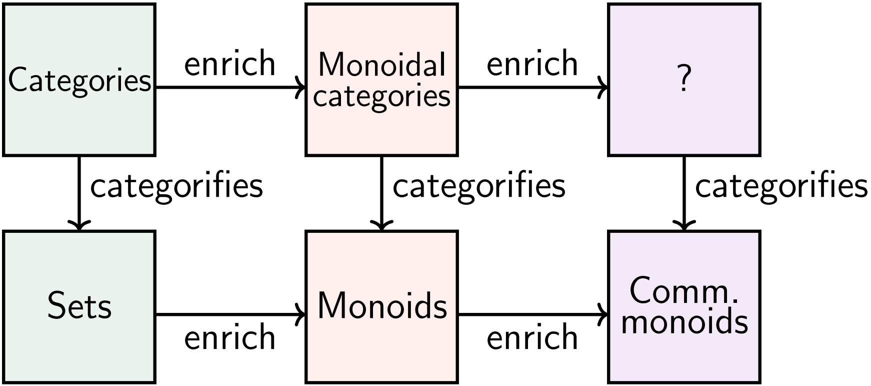
What are...braided categories?

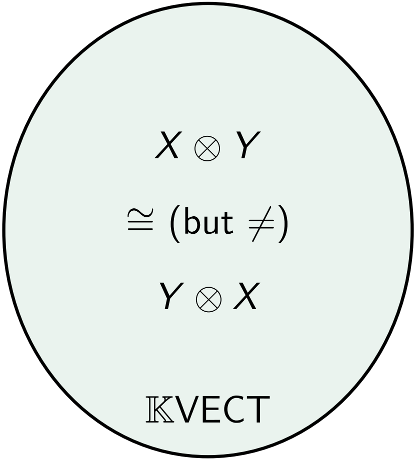
Or: Braids and categories

Filling in more question marks



- Monoidal categories categorify monoids
- Commutative monoids are enriched monoids
- What completes the rectangle?

Vector spaces again (what else?)

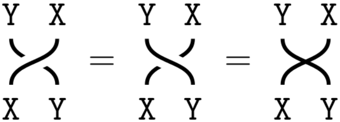


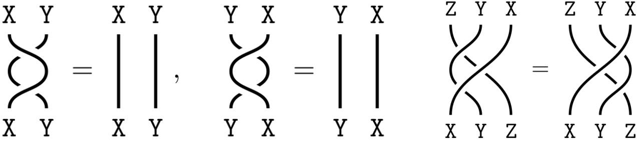
- In $\mathbb{K}$VECT we have $X \otimes Y \cong Y \otimes X$
- The isomorphism is the swap map $\beta_{X,Y} : x \otimes y \mapsto y \otimes x$
- This looks like commutativity

String diagram and braids

$\beta_{\mathtt{X},\mathtt{Y}} \rightsquigarrow$ 

, $\beta^{-1}_{\mathtt{X},\mathtt{Y}} \rightsquigarrow$ 

actually for $\mathbb{K}$VECT: $\beta_{\mathtt{X},\mathtt{Y}} = \beta^{-1}_{\mathtt{X},\mathtt{Y}} \rightsquigarrow$ 



- Denote the swap map by a crossing
- Categorically commutativity looks like a braid group action

For completeness: A formal definition

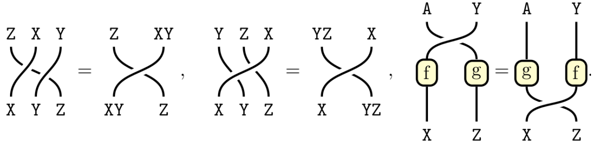
A monoidal category C is braided (strict) (extra data!) if:

- There exists a collection of natural isomorphism

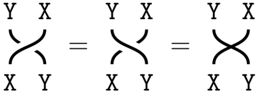
$$\beta_{X,Y} : XY \xrightarrow{\cong} YX, \text{ think } \beta_{X,Y} \leftrightsquigarrow$$


$$, \quad \beta^{-1}_{X,Y} \leftrightsquigarrow$$

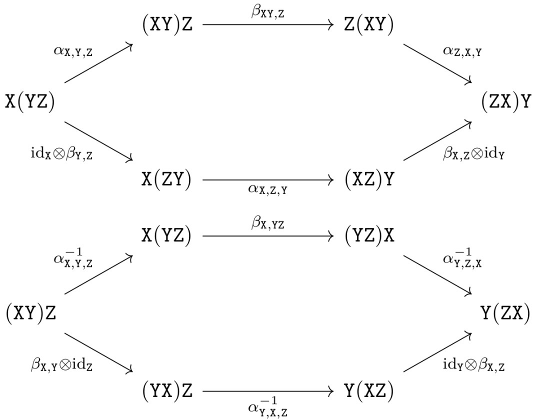

- We have naturality and sliding type relations (these imply the braid relations):



A category is symmetric (strict) if it is braided with

$$\beta_{X,Y} = \beta^{-1}_{X,Y}, \text{ think } \beta_{X,Y} = \beta^{-1}_{X,Y} \leftrightsquigarrow$$


Strict vs. not strict



- The non strict definition of braided involves several big commutative diagrams
- As usual, there is a strictification result

  Every braided category is braided equivalent to a strict braided category

  so I ignored the difference

Thank you for your attention!

I hope that was of some help.