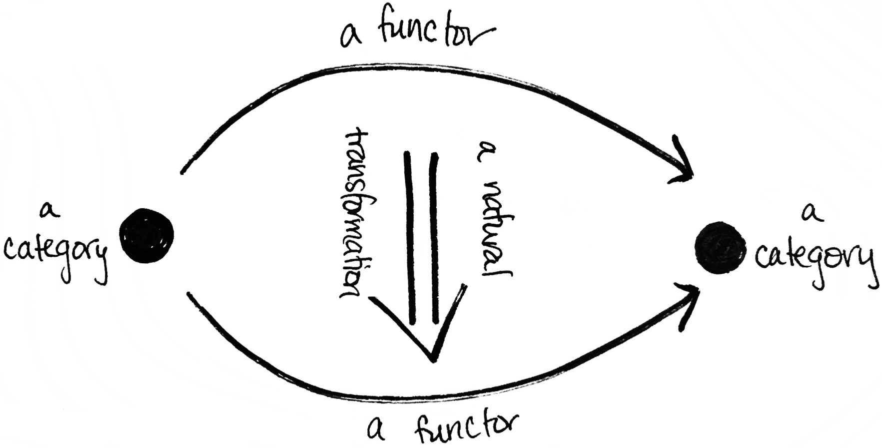
# What are...natural transformations?

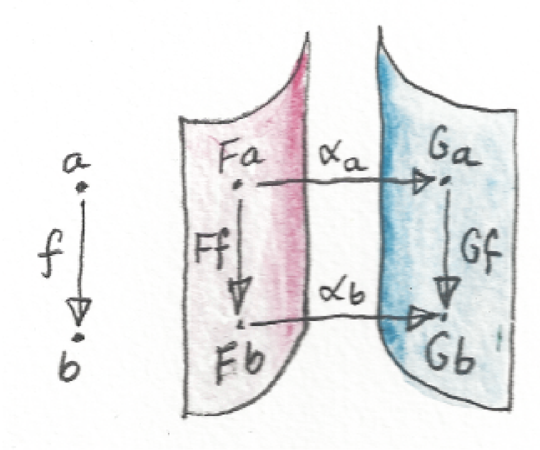
Or: Maps between functors

## The whole video on one slide



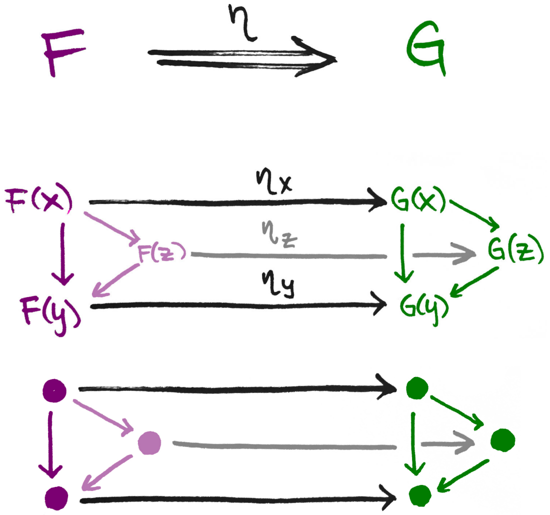

- 0dim Categories – Objects
- 1dim Functors – Arrows
- 2dim Natural transformations (nat trafo) – Arrows between arrows

## Connecting diagrams




- A nat trafo $\alpha\colon F \Rightarrow G$ should be the correct map between functors
- Functors associate Lines↦Lines
- $\alpha$ should associate Lines↦Squares

## Connecting fancier diagrams

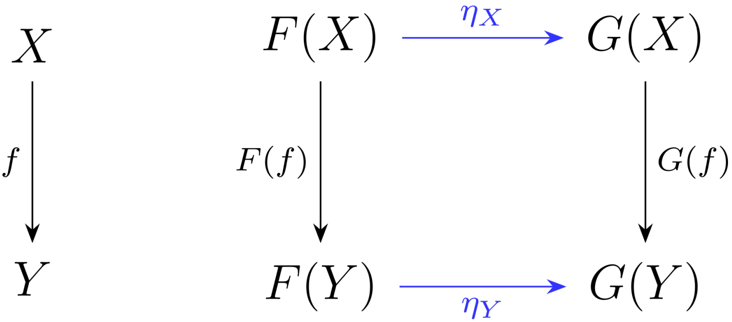



- Functor About commuting diagrams
- Nat trafo About commuting polytopes

## For completeness: A formal definition

A nat trafo $\eta\colon F \Rightarrow G$ is a mapping that:

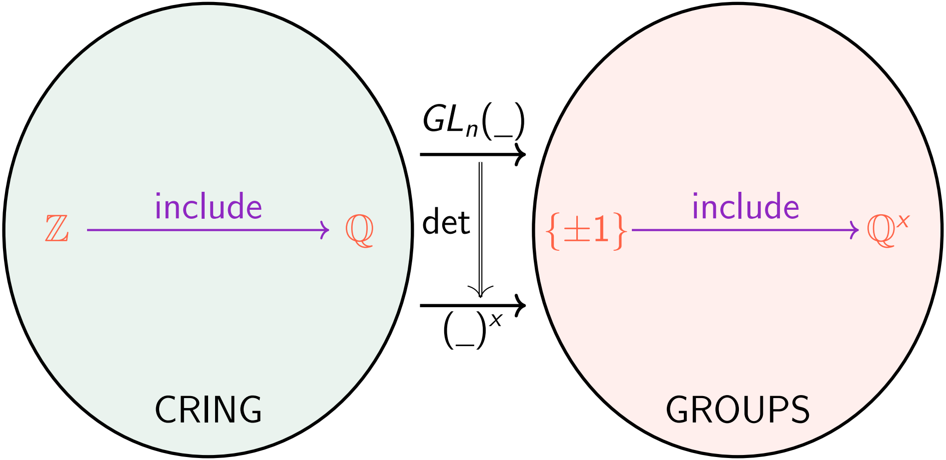
- associates each object $X$ in $C$ to an arrow $\eta_X\colon F(X) \to G(X)$ in $D$ Points↦Lines
- such that $\eta_Y F(f) = G(f)\eta_X$ Nat trafo square




Here $F, G\colon C \to D$ are functors with same source and target categories

The tip of the iceberg: the arrow between nat trafos is called modification

## The determinant is a nat trafo




- $GL_n(\_)$ and $(\_)^x$ (group of units) are functors from CRING to GROUPS.
- det: $GL_n(\_) \Rightarrow (\_)^x$ is a nat trafo
- Why? det is defined by the same formula for every ring, so $\det_S GL_n(f) = f \det_R$

**Thank you for your attention!**

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I hope that was of some help.