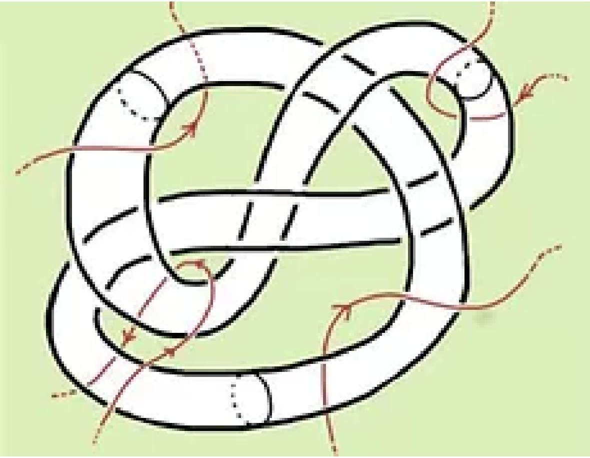
What is...Dehn surgery?

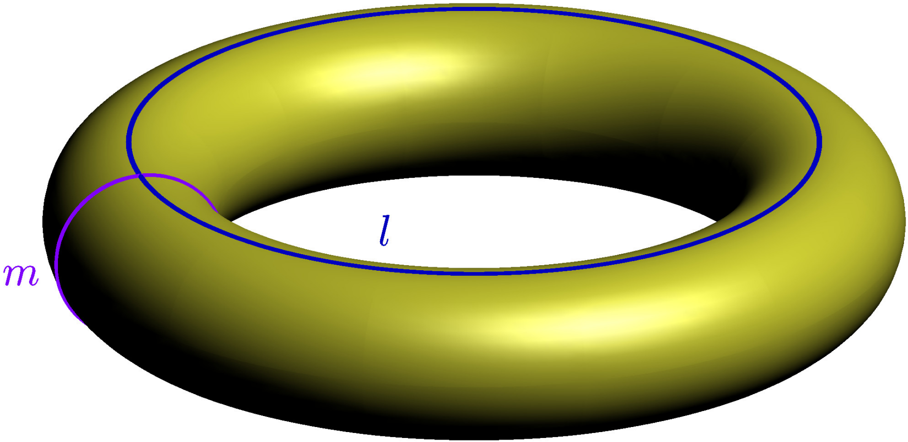
Or: Knots and three manifolds

Knot complements, again!



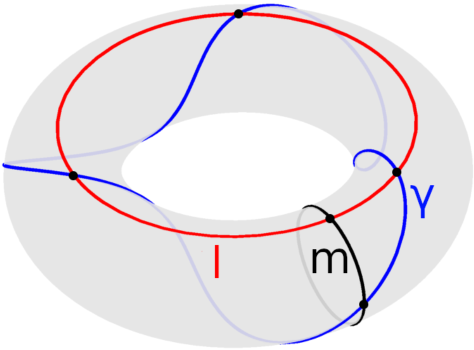
- A knot complement $S^3 \setminus \text{int}(K)$ is a 3mfd bounding a torus
- Idea Glue back in a solid torus ST, but "twisted"
- This should produce a closed 3mfd

The image of the meridian



- Any such gluing is determined by the image of the meridian m
- m goes to some simple closed curve γ in $T = \partial ST$, and it hence suffices to describe γ

Two numbers p, q



γ "=" $3m + l$

- Up to sign (=orientation) each simple closed curve γ in T is determined by how often p it follows the meridian m and how often q the longitude l
- If you like this: $[\gamma] = p \cdot [m] + q \cdot [l] \in H_1(\partial T)$
- Thus, every gluing of T is determined by $p, q \in \mathbb{Z}$
- The ratio $p/q \in \mathbb{Q} \cup \{\infty\}$ is the surgery coefficient

For completeness: A formal statement

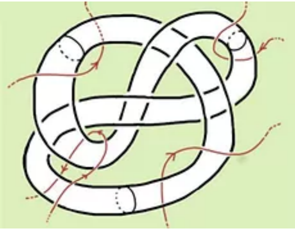
Every closed, orientable, connected 3mfd can be obtained by Dehn surgery, that is:

(i) Pick a finite collection of knots in S^3

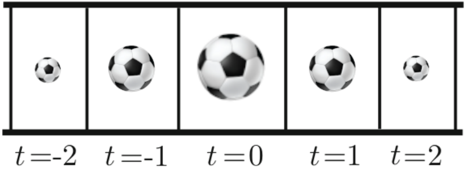
(ii) Pick a surgery coefficient for each knot

(iii) Perform the "remove-insert" surgery

- Example $1/0$-surgery gives S^3

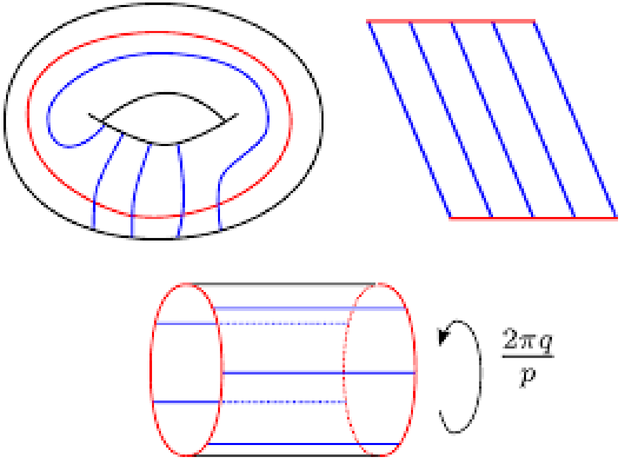


meridian to meridian $\longrightarrow$



- One can even restrict to integral coefficients $p/q \in \mathbb{Z} \cup \{\infty\}$

Lens spaces



- Example Surgery of S^3 along a p/q unknot gives the p/q Lens space
- In particular, all $1/q$ surgeries along unknots give back S^3
- Missing: some tool to tell whether the obtained 3mfds are the same

Thank you for your attention!

I hope that was of some help.