

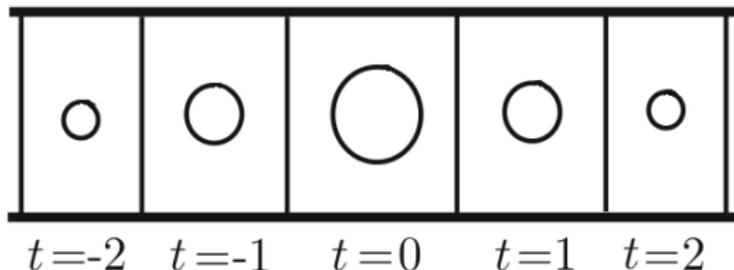
**What are...four manifolds?**

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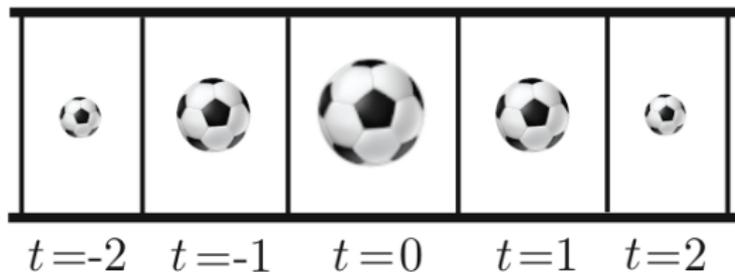
Or: Movies and colors!

## Two, three and then...?

A two mfd:

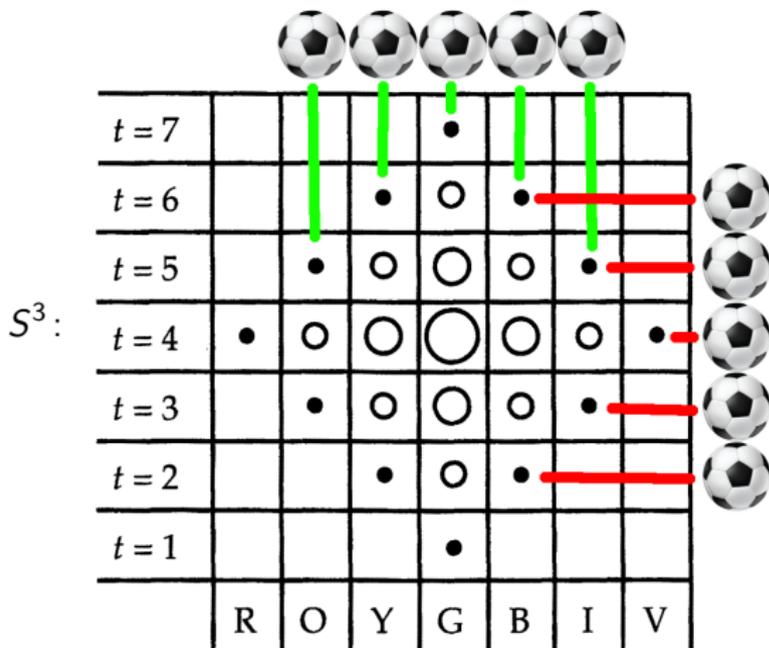


A three mfd:



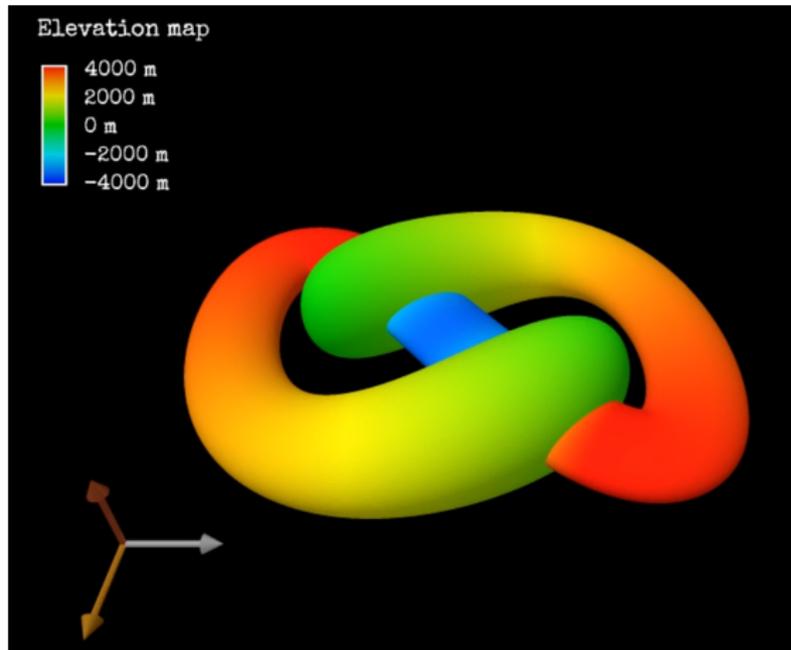
- ▶ 2 mfd's are classifiable and closed orientable 2 mfd's are spheres plus handles
- ▶ 3 mfd's are harder but closed orientable ones are given by knots-type-diagrams
- ▶ Question What about dimension four?

## Four manifolds are easy to define but...



- ▶ 2mfds are locally made of 2d balls=discs
- ▶ 3mfds are locally made of solid balls=bowling balls
- ▶ 4mfds are locally made of solid 4d balls=whatever is bound by  $S^3$

## Colors often help



- ▶ Using three coordinates and one color gives **four** dimensions
- ▶ **Above** A knotted  $S^2$  in  $\mathbb{R}^4$
- ▶ Maybe 4d is so fascinating because it is within reach but then not really

## For completeness: A formal definition

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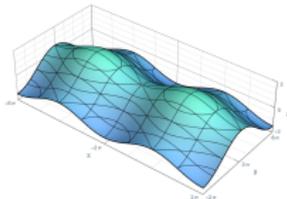
A closed 4mfd  $M$  is a topological spaces such that:

- (i) Every  $x \in M$  has an open neighborhood  $\cong$  to  $(X \subset \text{Euclidean 4-space})$  open 4d discs and  $M$  is compact
  - (ii)  $M$  is nonempty, second-countable, and Hausdorff Technical assumptions
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A 4mfd  $M$  with boundary is a topological spaces such that:

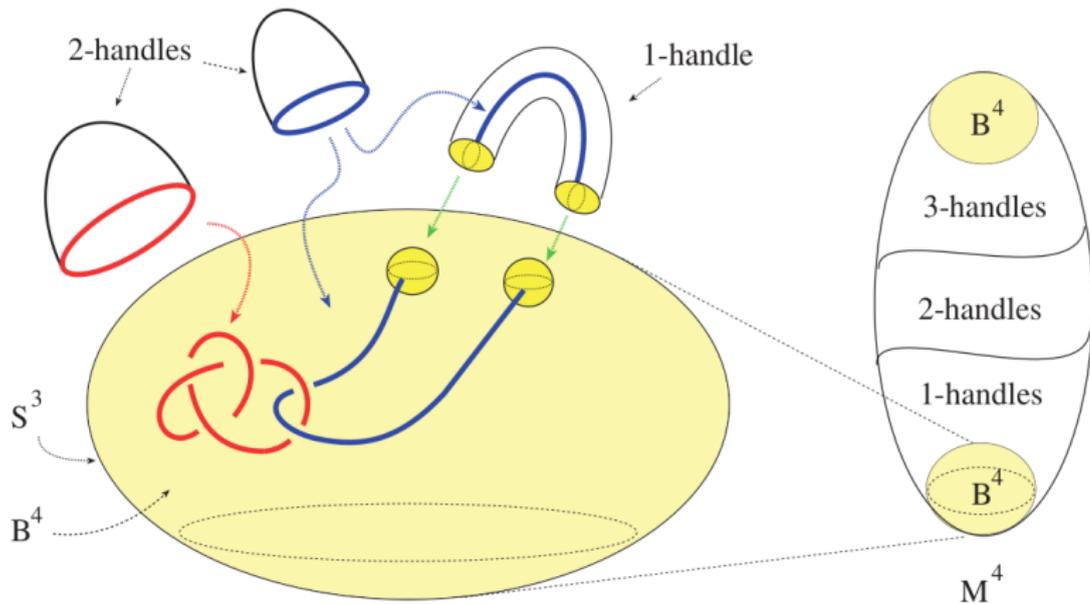
- ▶ Every  $x \in M$  has an open neighborhood  $\cong$  to  $(X \subset \text{closure of 4d half-space})$  open 4d discs or 4d half-discs
  - ▶  $M$  is nonempty, second-countable, and Hausdorff Technical assumptions
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Real surfaces are the first nontrivial example of real manifolds



Complex surfaces are the first nontrivial example of complex manifolds

## 4mfd and knots



► We have already seen Kirby calculus for 3mfd

► We will see Kirby calculus for 4mfd

**Thank you for your attention!**

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I hope that was of some help.