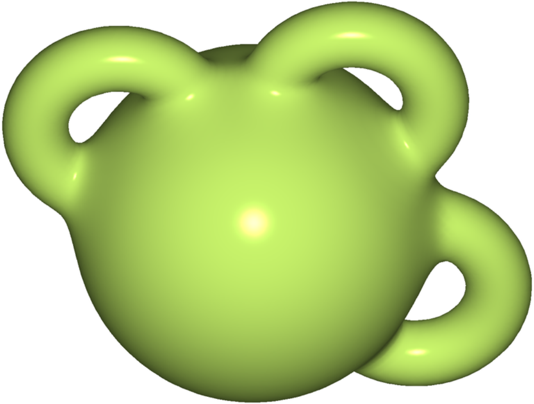
What is...a handle decomposition?

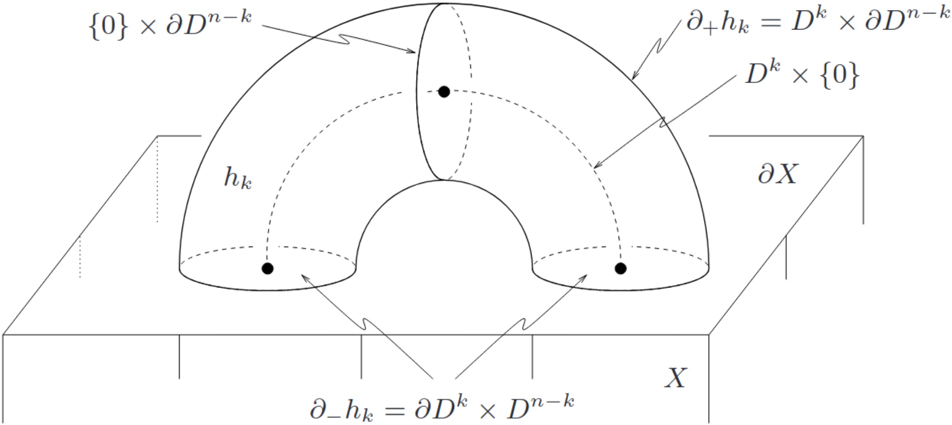
Or: Handles and even more handles!

Handle decomposition



- Handle decomposition $\leftrightsquigarrow$ attaching k handles to n mfds X
- k handle $= h^k = D^k \times D^{n-k}$
- Attaching $=$ gluing map $f\colon (\delta D^k) \times D^{n-k} \to \delta X$

A picture to keep in mind



- Attaching sphere $f\left((\delta D^k) \times \{0\}\right) \subset X$ Belt sphere $\{0\} \times (\delta D^{n-k}) \subset h^k$
- Handle decomposition $X = X_{-1} \cup X_0 \cup \ldots \cup X_n$ where each X_k is obtained from X_{k-1} by the attaching of k-handles

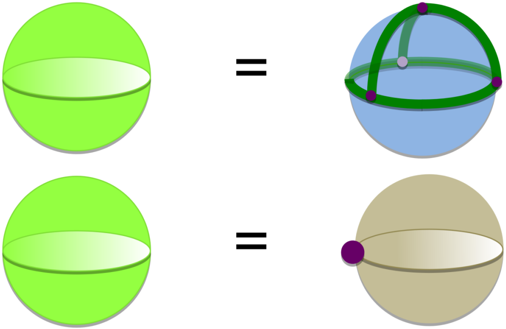
Heegaard is back

- Write a 3mdf $M = H \cup H'$ for handlebodies H, H' with $H \cap H' = \delta H = \delta H'$; this means M is glued together along H, H'
- This is called a Heegaard splitting
- The handlebodies here are h^1, so 1 handles

For completeness: A formal statement

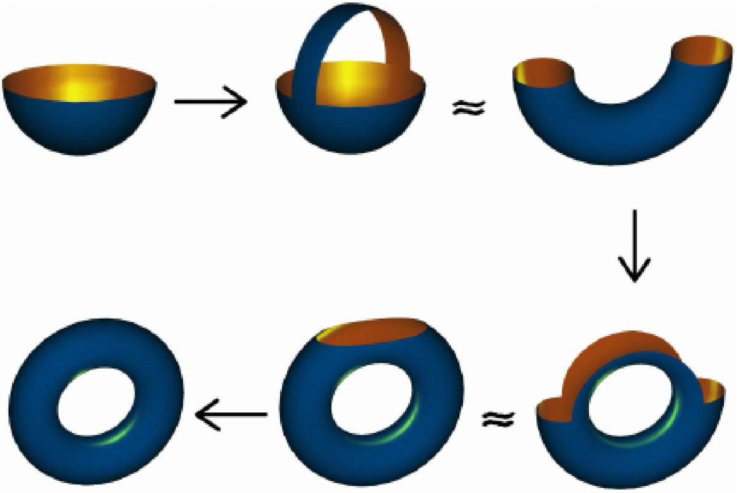
Any closed orientable (+reasonable adjectives) n-mfd admits a handlebody decomposition

- Reasonable adjectives: “anything” for $n \geq 6$, “piecewise linear” for other n
- Cell decompositions work well for topological spaces:



- Handle decompositions are the analog of cell decompositions for manifolds

Attaching handles in 2d



- In 2d we attach 0 handles, 1 handles and 2 handles
- 0 handle = cap, 1 handle = handle, 2 handles = cap

Thank you for your attention!

I hope that was of some help.