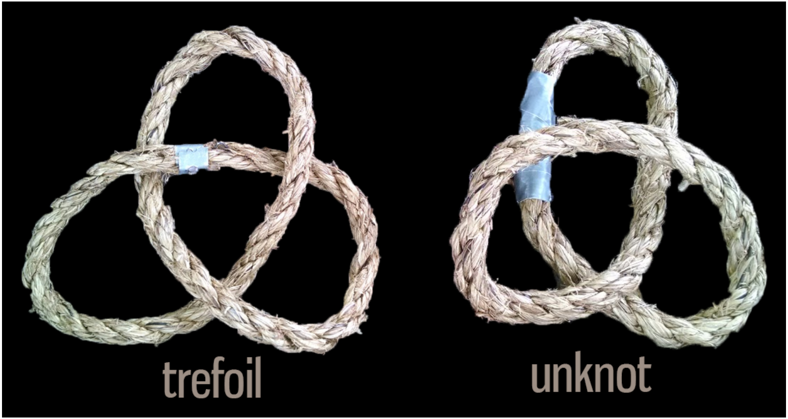
What is...a knot coloring?

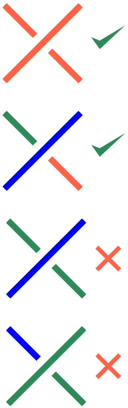
Or: A colorful approach

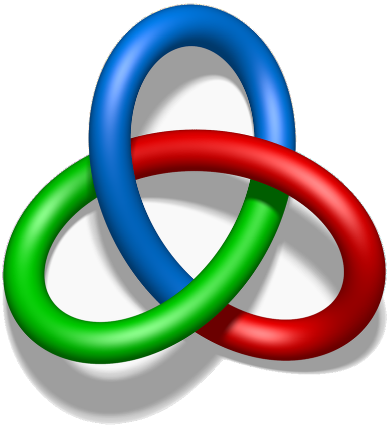
Obviously not!



- Question Is the trefoil trivial?
- Obviously not! Proof? By looking at it, or by building it from rope
- But what about a math proof? $\Rightarrow$ Knot invariant

Coloring projections

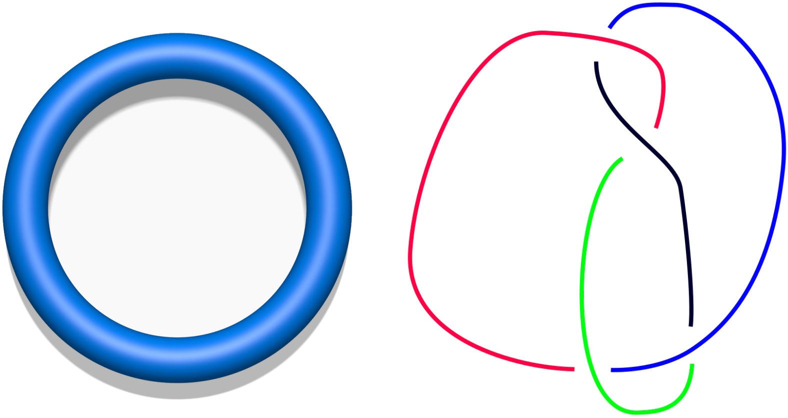




A projection is called tricolorable (red, green, blue) if it has a coloring with:

- At least two colors are used
- At each crossing, the three incident strands are either all the same color or all different colors

Some knots are not tricolorable

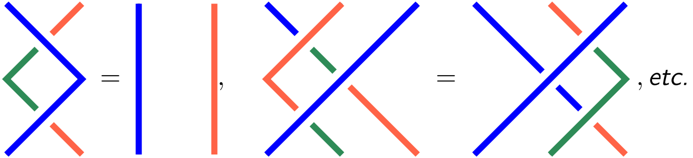


- Neither the unknot nor the figure eight knot are tricolorable
- **Question** What can tricolorability tell us about knots
- Right now it should actually be “Neither of the two projections is tricolorable”

For completeness: A formal statement

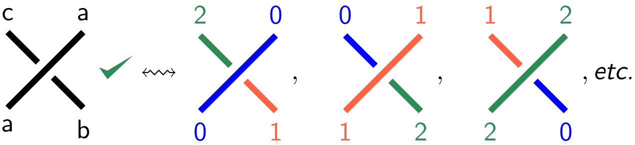
Tricolorability is a knot invariant

- In particular, trefoil $\neq$ unknot or figure eight
- The proof fits into one line:

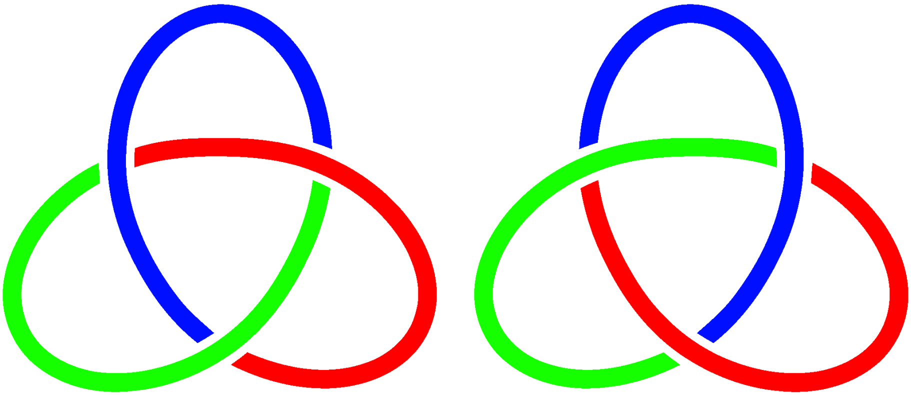


- There is also an n-coloring for n odd using the crossing condition

$$2a \equiv b + c \bmod n$$



Left = right-handed trefoil? No idea...



- The left-handed trefoil is tricoloralbe
- The right-handed trefoil is tricoloralbe
- Thus, we can't tell them apart

Thank you for your attention!

I hope that was of some help.