

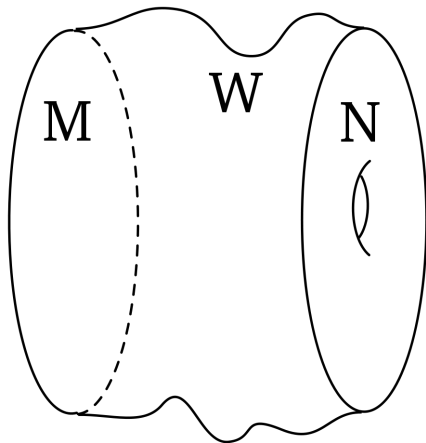
**What is...the h-cobordism theorem?**

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Or: Nice manifolds between manifolds

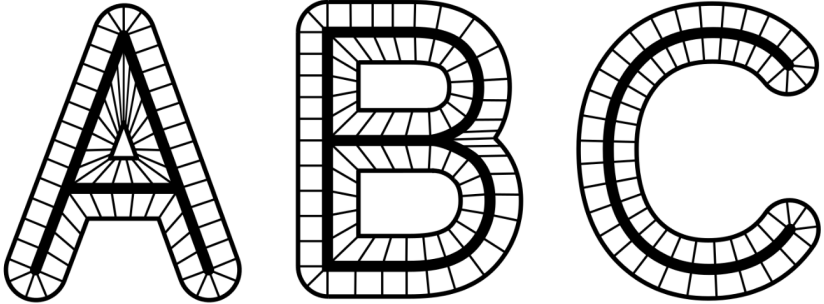
## Cobordisms

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- ▶ We want a **version of a map** between  $n$ mfd  $M$  and  $N$
  - ▶ A **cobordism**  $W$  between  $M$  and  $N$  is a  $(n + 1)$ mfd with boundary  $M \cup N$

## Homotopy equivalence



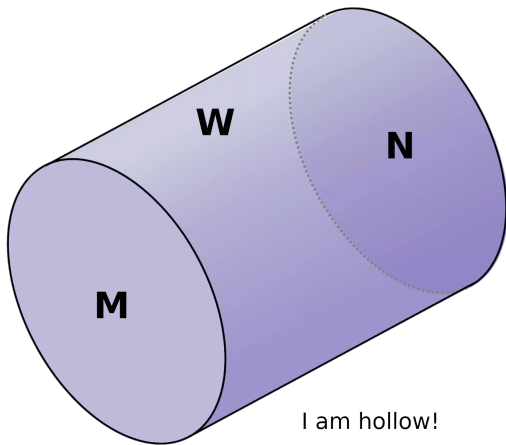
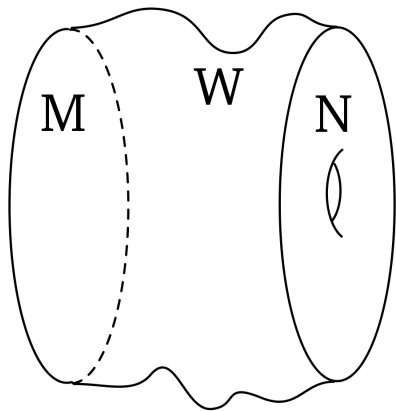
Homotopy types of the graphs underlying the alphabet:

Genus 0	Genus 1	Genus 2
CEFGHIJKLMNSTUVWXYZ	ADOPQR	B

- ▶ For this video, homotopy equivalence = we can continuously squeeze
- ▶ Homotopy equivalence is weaker than being homeomorphic
- ▶ In particular, homotopy equivalence can **jump between dimensions**

$h$ =homotopy equivalence

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- ▶  $h$ -cobordisms =  $W: M \rightarrow N$  so that  $M, N \hookrightarrow W$  are homotopy equivalences
  - ▶ **Left** Not an  $h$ -cobordism
  - ▶ **Right** An  $h$ -cobordism

# For completeness: A formal statement

Assume that:

- (i)  $M, N$  compact simply-connected orientable  $n$ mfds with  $n \geq 5$
- (ii)  $M, N$  are  $h$ -cobordant via  $W$  with  $W$  simply-connected

Then  $M$  and  $N$  are homeomorphic

- ▶ The magic is the **step** from homotopy equivalence to homeomorphism
- ▶ Smale got the field medal for this discovery

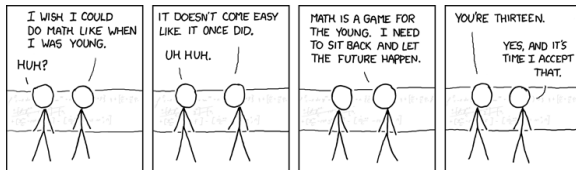
ANNALS OF MATHEMATICS  
Vol. 74, No. 2, September, 1961  
Printed in Japan

## GENERALIZED POINCARÉ'S CONJECTURE IN DIMENSIONS GREATER THAN FOUR

BY STEPHEN SMALE\*

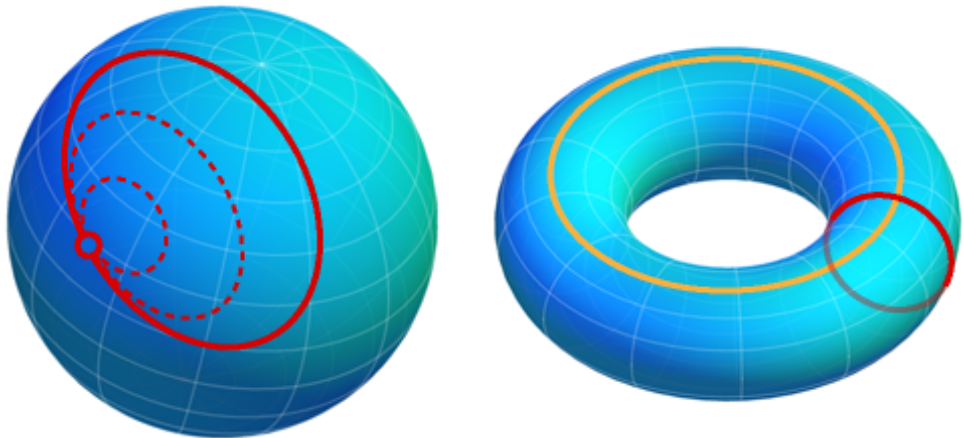
(Received October 11, 1960)

(Revised March 27, 1961)



## The Poincaré conjecture

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- ▶ Smale's breakthrough proved the Poincaré conjecture in  $\dim \geq 5$
  - ▶ We will discuss the proof in the next video

**Thank you for your attention!**

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I hope that was of some help.