# What is...the unknotting problem?

Or: How to detect the unknot?!

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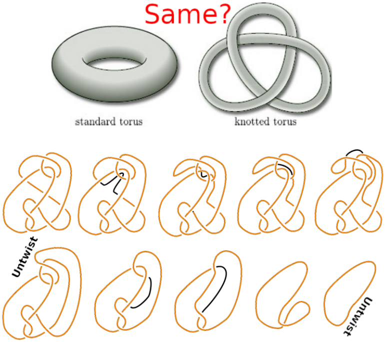
**Direction two – unknotting problem**

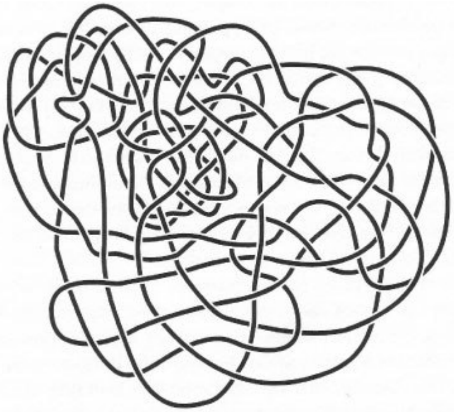



- Knot theory studies embedded mfds up to a different notion than homeomorphism
- One of the main open problems in knot theory: detect the unknot

- Unknotting problem Can we decide whether a given knot is the unknot
- This is one of the most crucial problems in knot theory
- Today I will show you some ways to 'solve' this question

## There are arbitrary hard unknots

Unknot:



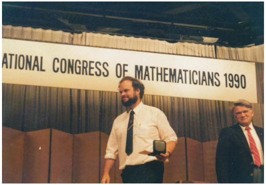
- Demon = diagram of the unknot where one can apply no simplifying moves
- Theorem There are infinitely many demons
- Unknotting problem Decide whether a knot diagram is the unknot

# The Jones polynomial (reminder)

(i) $\langle \emptyset \rangle = 1$ Normalization

(ii) $\langle \bigcirc \cup L \rangle = -(A^2 + A^{-2}) \cdot \langle L \rangle$ Pulling out circles

(iii) Kauffman Skein




- The Jones polynomial (JP) is a powerful and simple invariant of knots
- Jones got the fields medal for the discovery
- Conjecture JP(knot)=JP(unknot) $\Leftrightarrow$ knot=unknot

# For completeness: A formal statement

Khovanov homology (Kh) categorifies the Jones polynomial and:

Kh(knot)=Kh(unknot) $\Leftrightarrow$ knot=unknot

- Khovanov homology is 'reasonably easy' to compute (roughly in $2^n$ time steps where $n$=number of crossings)
- This 'solves' the unknotting problem
- Kronheimer–Mrowka gave a beautiful ICM talk about this (and related) breakthrough(s) Google 'Kronheimer Mrowka ICM 2018'

Detecting knottedness with $Kh(K)$

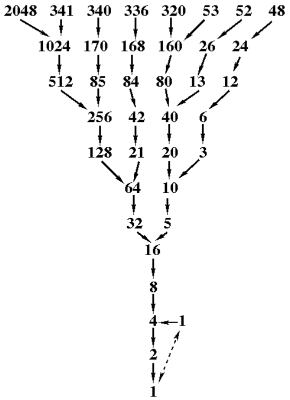
**Corollary:** If $K$ is non-trivial then (with $\mathbb{Z}/2$ coefficients),

$$\dim Kh(K) > 2$$

*"Khovanov homology is an unknot-detector"*

## Why is Kh successful while JP fails?

Collatz conjecture:




- The JP probably also detects the unknot but we cannot show this as there is not enough structure
- “Not enough structure” happens very often – think about the many conjectures about number sequences‘
- Kh is richer and that is very helpful to prove the result

**Thank you for your attention!**

I hope that was of some help.