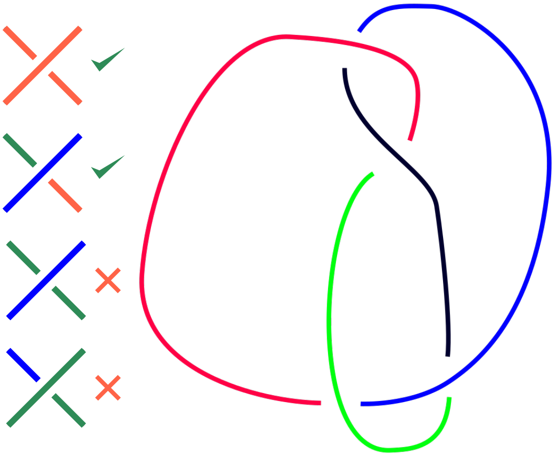
What is...the knot determinant?

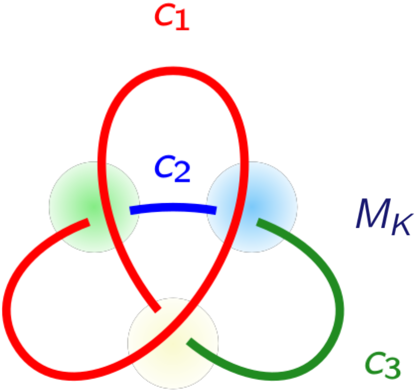
Or: Enter, linear algebra

Knot colorings



- Colorability is an intuitive and good knot invariant
- Problem A priori it is not easy to decide whether a knot is n-colorable
- Idea Linear algebra should give an algorithm to decide n-colorabtlity

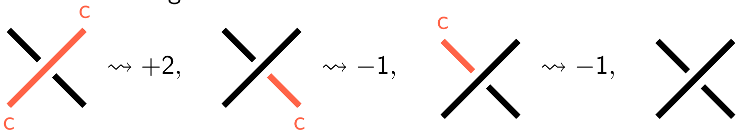
A matrix for a projection



$$M_K = \begin{array}{c} \\ \end{array}\begin{array}{ccc} c_1 & c_2 & c_3 \\ \end{array}$$

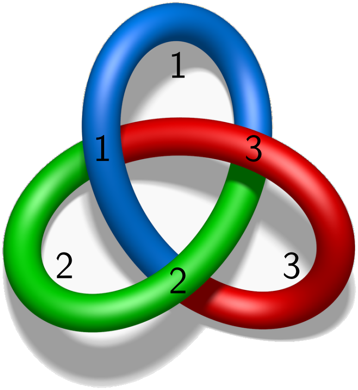
$$M_K = \begin{bmatrix} 2-1 & -1 & 0 \\ 2 & -1 & -1 \\ 2-1 & 0 & -1 \end{bmatrix} \quad \text{(columns } c_1, c_2, c_3\text{)}$$

- Form a matrix M_K with # crossings rows and # segments columns
- Contribution of segment c



$\rightsquigarrow +2,\quad \rightsquigarrow -1,\quad \rightsquigarrow -1,\quad \rightsquigarrow 0$

Knot determinant



$\rightsquigarrow M_K =$

2	-1	-1
-1	2	-1
-1	-1	2

det of the projection: 3

- The determinant of M_K-one row/column is called the knot determinant
- Note that the determinant depends on the projection

For completeness: A formal statement

The knot determinant of some projection is divisible by n odd

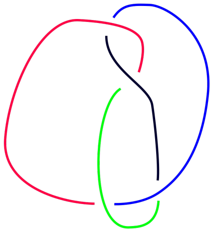
$\Leftrightarrow$

the knot determinant of any projection is divisible by n odd

$\Leftrightarrow$

the knot is n-colorable

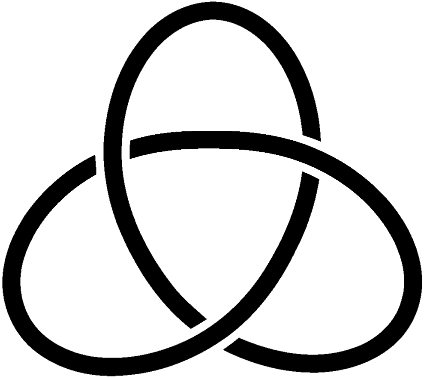
- This gives an algorithm to check n-colorability
- Example The figure eight knot is only 5-colorable:



$$\rightsquigarrow M_K = \begin{pmatrix} 2 & -1 & -1 & 0 \\ -1 & 0 & 2 & -1 \\ -1 & -1 & 0 & 2 \\ 0 & 2 & -1 & -1 \end{pmatrix} \rightsquigarrow \det = 5$$

$red = 0, blue = 2, green = 1, black = 3$

Left = right-handed trefoil? No idea...



- The left-handed trefoil has matrix $\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$, so det=3
- The right-handed trefoil has matrix $\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}$, so det=3
- Thus, we still can't tell them apart; for no *n*

Thank you for your attention!

I hope that was of some help.