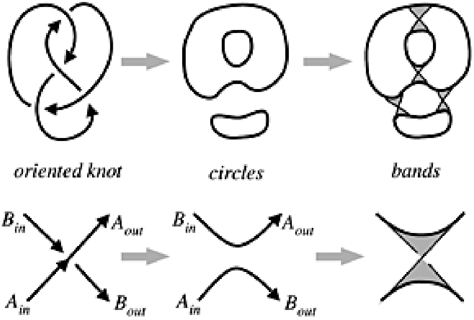
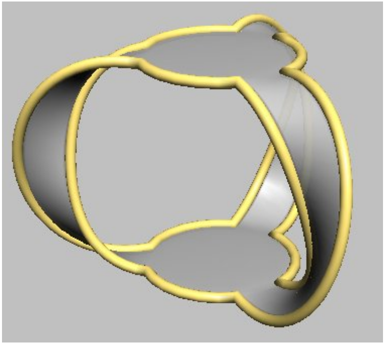
What is...the knot genus?

Or: Minimal surfaces and knots

Seifert's algorithm



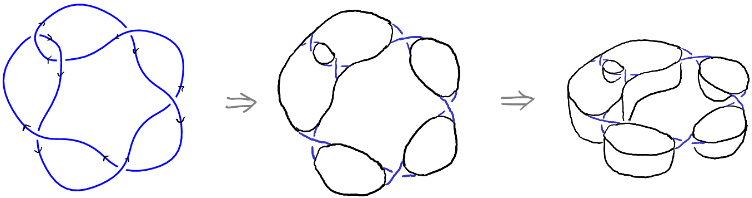


- For a given (oriented) knot diagram:
  - Replace all crossings by smoothings as above
  - Obtain discs
  - Connect the discs by bands as above
- Seifert's algorithm gives a surface bounding our knot

Minimal surfaces

- These Seifert surfaces are minimal area while bounding the knot
- These arise via soap films
- This "proves" they exist

The genus of a knot – almost



genus=1

genus=1

- $m = \#$ components, $d = \#$ crossings, $f = \#$ circles
- The genus of a knot projection is

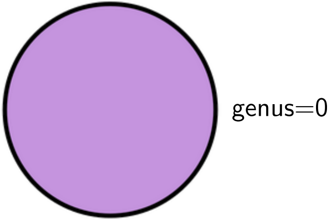
$$g = \frac{1}{2}(2 + d - f - m)$$

For completeness: A formal statement

Define the genus of a knot as the minimum of g over all projections

The genus is a knot invariant

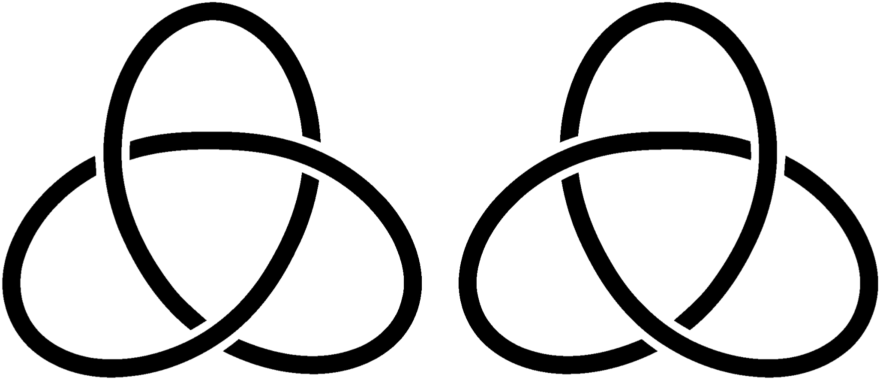
- Warning Seifert's algorithm for a fixed diagram might not give the minimal answer
- Genus=0 $\Leftrightarrow$ the knot is trivial



Warning: this fails for links

- The degree of the Alexander polynomial is a lower bound for 2xgenus (knots only)

Left = right-handed trefoil? No idea...



- The left-handed trefoil has genus one
- The right-handed trefoil has genus one
- Thus, we still can't tell them apart

Thank you for your attention!

I hope that was of some help.