What is...the bracket polynomial?

Or: Jones and co.

Kauffman skein calculus

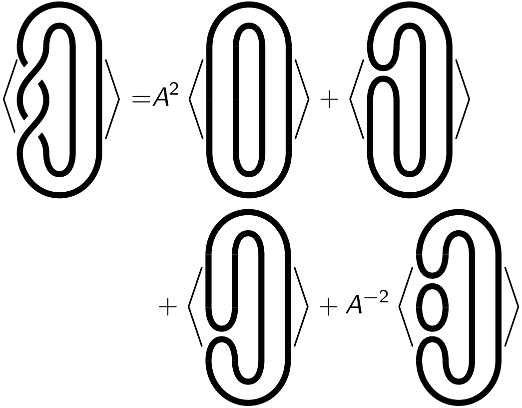
(i) $\langle \emptyset \rangle = 1$ Normalization

(ii) $\langle \bigcirc \cup L \rangle = -(A^2 + A^{-2}) \cdot \langle L \rangle$ Pulling out circles

(iii) Kauffman Skein

- The bracket polynomial $\langle \_ \rangle$ is a polynomial associated to a knot projection
- It is defined using a linear relation
- The linear relation involves the three ways to connect four points

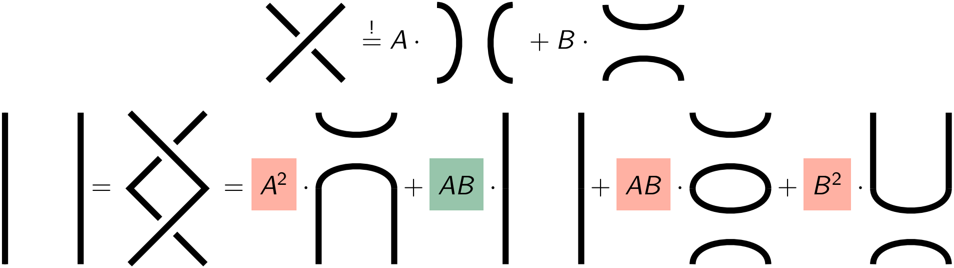
On the back of an envelope



$$=A^2(-(A^2+A^{-2}))^2+2(-(A^2+A^{-2}))+A^{-2}(-(A^2+A^{-2}))^2$$

- The definition of $\langle \_ \rangle$ gets rid of all crossings
- Computing $\langle \_ \rangle$ is therefore easy
- This is not recursive as the calculation of the Alexander–Conway polynomial

A unique solution



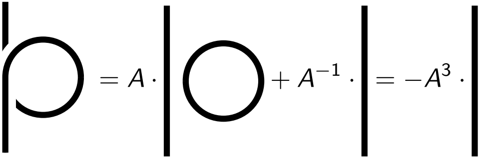
$AB \cdot id \Rightarrow A = B^{-1}$

$\text{Rest} \Rightarrow A^2 + A^{-2} = -\text{circle}$

- Idea there should be a relation among the three ways to connect four points
- Playing with Reidemeister moves gives a unique solution
- We get an invariant by construction

For completeness: A formal statement

The bracket polynomial $\langle \_ \rangle \in \mathbb{Z}[A, A^{-1}]$ is a knot invariant up to Reidemeister I:

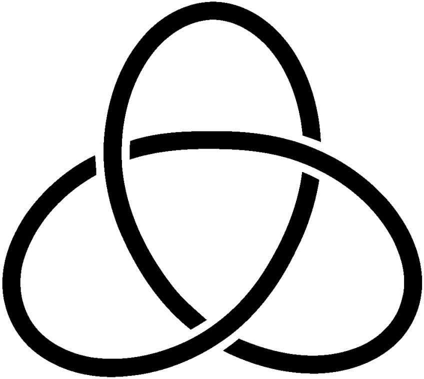


Hence:

Appropriately rescalled, $\langle \_ \rangle$ is a knot invariant

- A scaling of $\langle \_ \rangle$ (and changing variables $q = A^2$) gives the **Jones polynomial**
- The Jones polynomial changed knot theory drastically
- Among other things, Vaughan Jones was awarded the fields medal in 1990 for the discovery of the Jones polynomial

Left = right-handed trefoil? No!



- The left-handed trefoil has Jones polynomial $-q^4 + q^3 + q$
- The right-handed trefoil has Jones polynomial $-q^{-4} + q^{-3} + q^{-1}$
- Thus, they are different

Thank you for your attention!

I hope that was of some help.