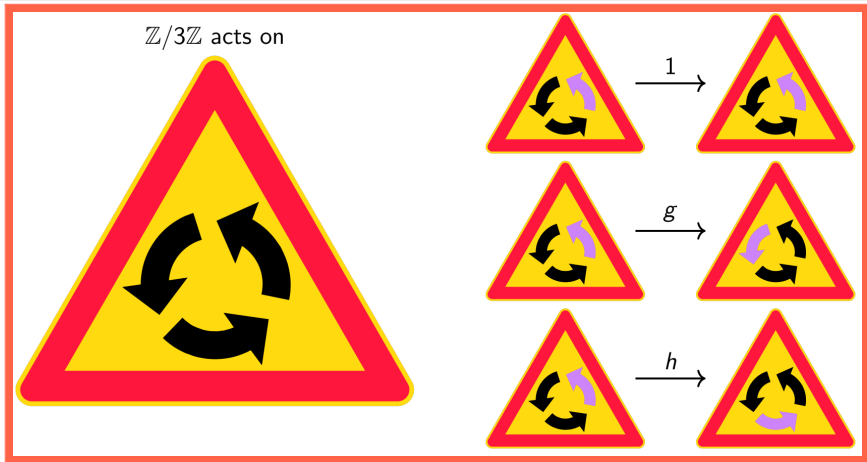


Lie theory - part 15

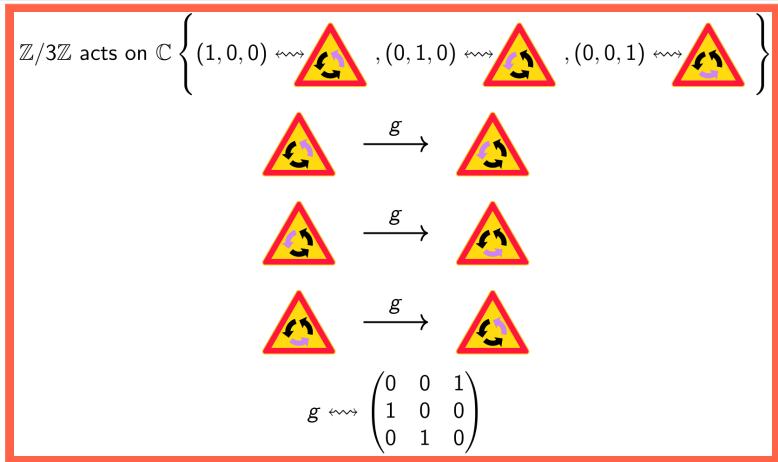
Or: Representations of Lie groups: first principles

Why representations?



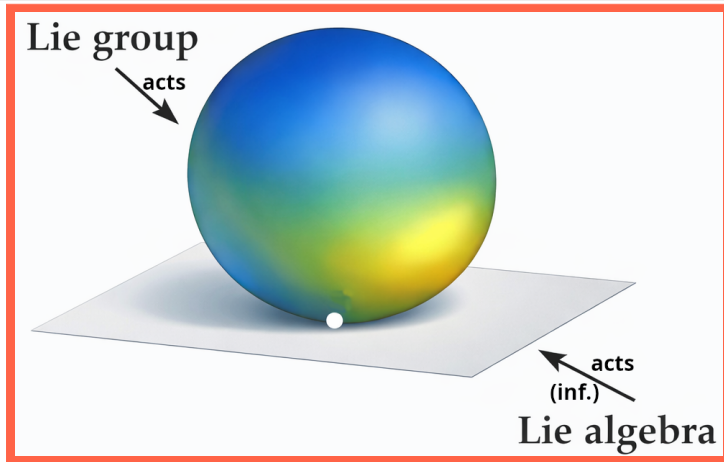
- ▶ **Symmetry** A Lie group is often too abstract until it acts on something
- ▶ **Linear world** Vector spaces are where we can compute
- ▶ **Main idea** A representation turns symmetry into linear motion

What is the data?



- ▶ **Each element** A symmetry $g \in G$ gives a matrix $\rho(g): V \rightarrow V$
- ▶ **Compatibility** Multiplication is respected: $\rho(gh) = \rho(g)\rho(h)$, $\rho(e) = \text{id}_V$
- ▶ **Dictionary** The whole group law is translated into matrix multiplication

Examples before definitions bite



- ▶ **Matrices** Matrix groups already act on column vectors by definition
- ▶ **Rotations** Rotations act on the plane or on space, not just on themselves
- ▶ **Permutations** Reordering coordinates is also a perfectly good linear action

Storytime: finite groups

The first ever published character table?

... der Ordnung 2 bilden eine zweisellige Classe (1), die
Ordnung 3 zwei inverse Classen (2) und (3) = (2'). Sei ρ eine prin-
zipielle Wurzel der Einheit.

Tetraeder. $h = 12$.

	$\chi^{(0)}$	$\chi^{(1)}$	$\chi^{(2)}$	$\chi^{(3)}$	h_α
χ_0	1	3	1	1	1
χ_1	1	-1	1	1	3
χ_2	1	0	ρ	ρ^2	4
χ_3	1	0	ρ^2	ρ	4

Die Werthe von χ_0 sind zugleich die von $f = e$.

- ▶ Frobenius' character table of A_4 ~1896
- ▶ Character tables were around since the beginning of rep theory
- ▶ They contain basically all info about group reps in an efficient way

- ▶ Frobenius + friends In the 1890s, characters turned finite groups into numbers
- ▶ Importance A bit later, characters were a crucial part of finite group theory
- ▶ Key (in hindsight) Finite group reps are combinatorial characters

Storytime: Lie groups

The Classical Groups: Their Invariants and Representations is a mathematics book by [Hermann Weyl](#) published in 1939. The book describes classical [invariant theory](#) in terms of [representation theory](#). It is largely responsible for the revival of interest in invariant theory, which had been almost killed off by [David Hilbert's](#) solution of its main problems in the 1890s.

The second edition was published in 1946. Weyl gave an informal talk about the topic of his book in 1939.

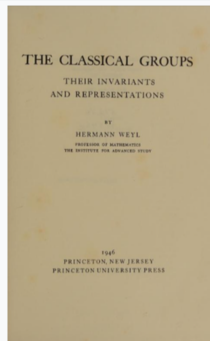
Reception [\[edit \]](#)

Roger Howe called the book "wonderful and terrible".

References [\[edit \]](#)

- Howe, Roger (1988), "[The classical groups and invariants of binary forms](#)" [↗](#), in [Wells, R. O. Jr.](#) (ed.), *The mathematical heritage of Hermann Weyl* (Durham, NC, 1987), Proc. Sympos. Pure Math., vol. 48, Providence, R.I.: [American Mathematical Society](#), pp. 133–166 [↗](#), ISBN 978-0-8218-1482-6, MR 0974333 [↗](#)
- Howe, Roger (1989), "Remarks on classical invariant theory.", *Transactions of the American Mathematical Society*, **313** (2), American Mathematical Society: 539–

The Classical Groups



Author	Hermann Weyl
Publication date	1939
ISBN	9780691057569

- ▶ **Weyl + friends** In the 1920s, Lie groups entered representation theory in force
- ▶ **Importance** A bit later, Lie groups became a kind of Fourier analysis
- ▶ **Key (in hindsight)** Lie group reps are geometric (e.g. root system) characters

Thank you for your attention!

Next time: Representations of Lie algebras