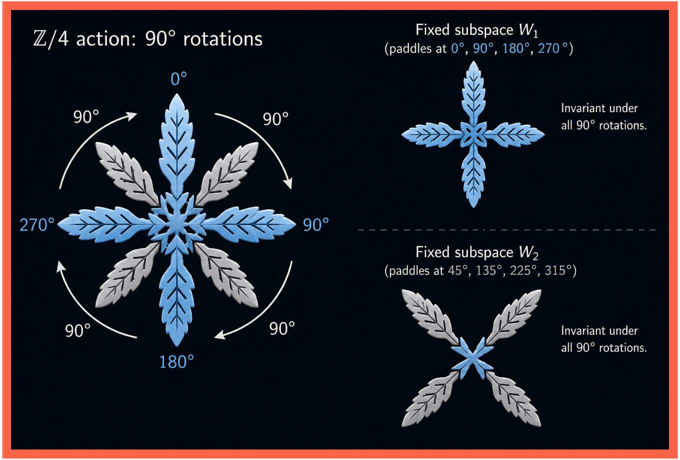
Lie theory - part 17

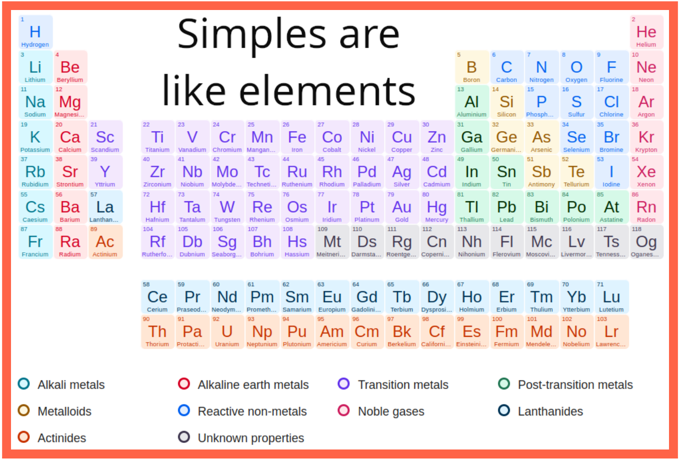
Or: Simple vs reducible

Why split a representation?



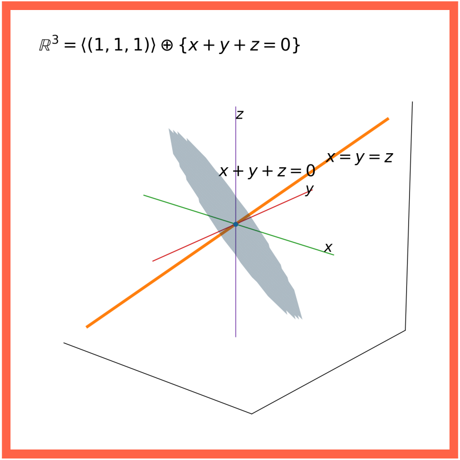
- **Idea** A complicated action may secretly be several simpler actions side by side
- **Invariant pieces** Some subspaces stay put under every symmetry in sight
- **Goal** Find the basic building blocks before asking for classification

Reducible vs irreducible



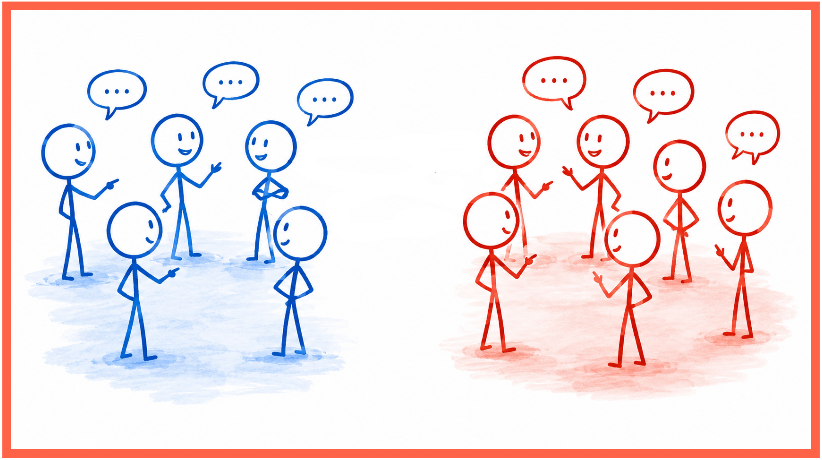
- **Reducible** Nontrivial invariant subspace $\Rightarrow$ the action has an internal seam
- **Simple** No such seam $\Rightarrow$ the representation behaves like one indivisible piece
- **Example** Diagonal matrices acting on $\mathbb{C}^2$ preserve the two coordinate lines

A favorite example: three coordinates



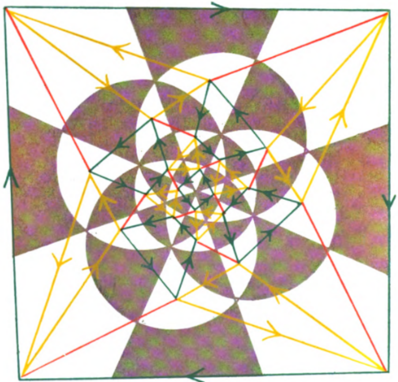
- Symmetry Permuting three coordinates acts on $\mathbb{C}^3$
- $\oplus$ The all-equal line stays fixed, while the sum-zero plane moves around
- In formulas $\mathbb{C}^3 \cong \mathbb{C} \oplus \mathbb{C}^2$ as S_3-reps

Schur's lemma, in human language



- Rigidity Maps between simples have remarkably little room to manoeuvre
- Consequence Different simples do not talk to each other
- Why care This turns allows us to compare, count, and eventually classify

Decomposition as a strategy



From Burnside's 1911 book

The octohedral group represented as a colour-group

- Toolkit Break reps into pieces, understand the pieces, then rebuild the whole
- Storytime Averaging made this philosophy work beautifully for finite groups
- Next bridge Compact Lie groups inherit a similar miracle

Thank you for your attention!

Next time: Compact groups and complete reducibility