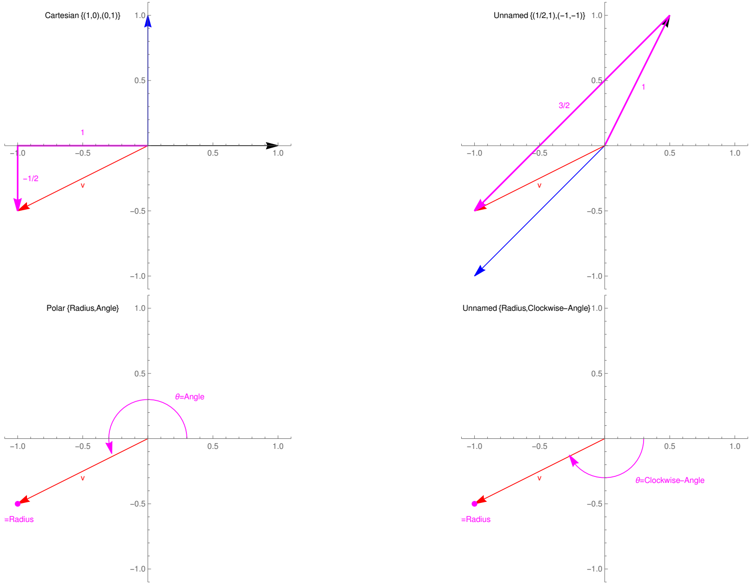
What is...a basis?

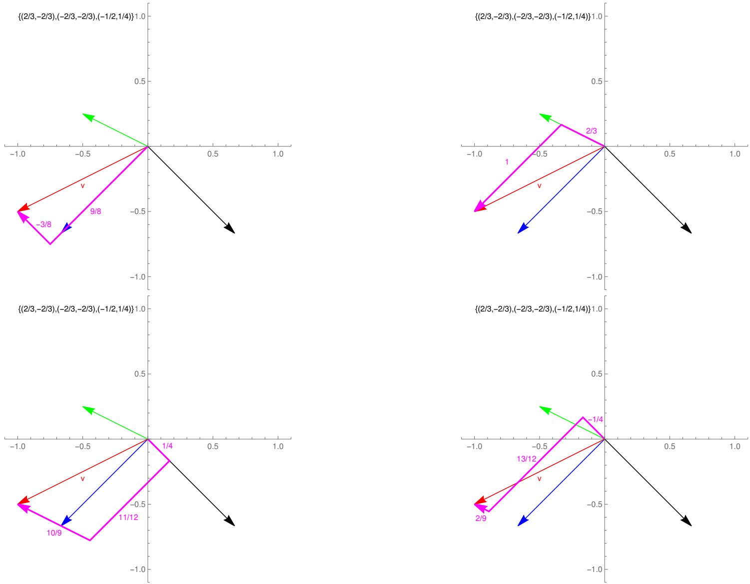
Or: A notion of dimension.

Ways to write a vector.



All of these are equally valid.

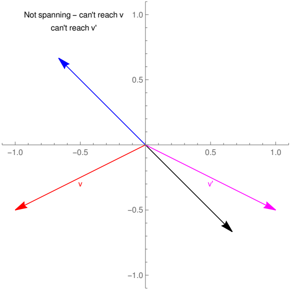
Linear dependent – too much information

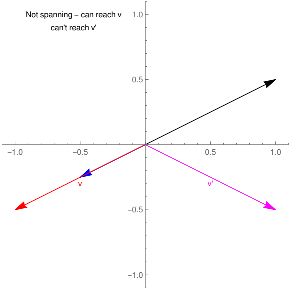


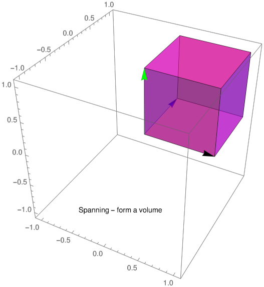
With three linearly independent vectors we can write v in infinitely many ways.

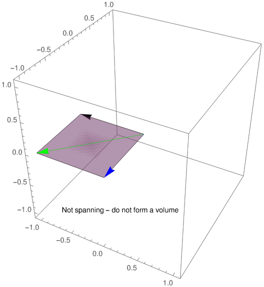
Choice, bad!

Not spanning – not enough information









With too few or "badly positioned" vectors we might not be able to write v at all.

Clearly bad!

For completeness: A formal definition.

Let $B = \{v_1, ..., v_n\}$ be a subset of some vector space V

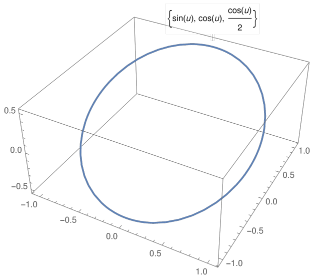
- B is called linearly independent if $\sum \lambda_i v_i = 0$ has only the trivial solution $\lambda_i = 0$
- B is called spanning if every $v \in V$ can be written as $v = \sum \lambda_i v_i$ for some $\lambda_i \in \mathbb{K}$
- If B is both, then B is called a basis

Important facts:

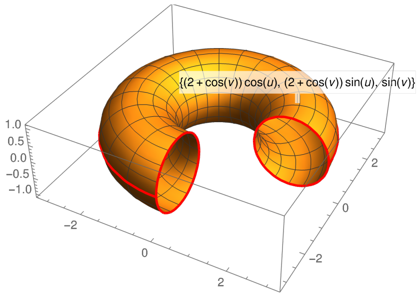
- If B is a basis, then every vector $v \in V$ can be uniquely written as $v = \sum \lambda_i v_i$ for some $\lambda_i \in \mathbb{K}$
- Two bases always have the same size, the dimension of V

Dimensions need not to be linear

One coordinate
determines it
dimension=1



Two coordinates
determine it
dimension=2



Thank you for your attention!

I hope that was of some help.