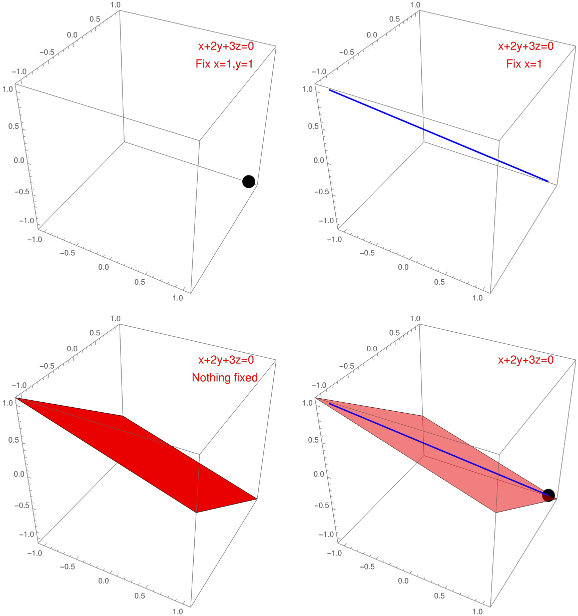
What is...a system of linear equations?

Or: Intersections matter.

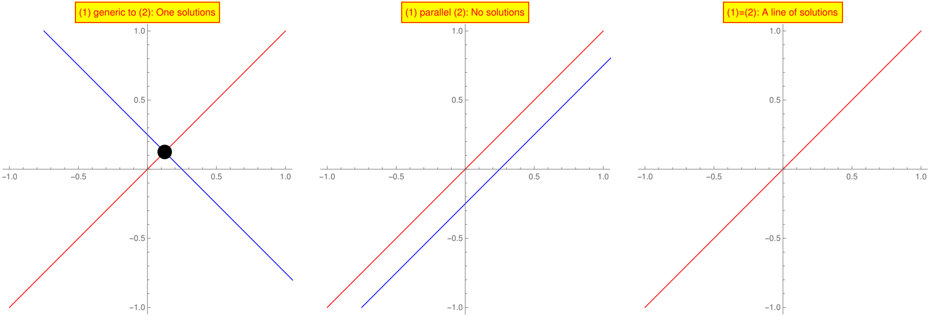
A linear equation is a linear space

$$1 \cdot x + 2 \cdot y + 3 \cdot z = 0 \rightsquigarrow \text{plane in } \mathbb{R}^3$$



Let us analyze dim 2

$$\begin{cases} ax + by = c & (1) \\ dx + ey = f & (2) \end{cases}$$



The left is the generic case – it almost always happens

How to solve these?

$$\begin{cases} 1x + 2y = 3 & (1) \\ 4x + 5y = 6 & (2) \end{cases} \leftrightsquigarrow \left(\begin{array}{cc|c} 1 & 2 & 3 \\ 4 & 5 & 6 \end{array}\right)$$

$$\begin{aligned} \left(\begin{array}{cc|c} 1 & 2 & 3 \\ 4 & 5 & 6 \end{array}\right) &\sim \left(\begin{array}{cc|c} -4 & -8 & -12 \\ 4 & 5 & 6 \end{array}\right) \sim \left(\begin{array}{cc|c} -4 & -8 & -12 \\ 0 & -3 & -6 \end{array}\right) \\ &\sim \left(\begin{array}{cc|c} -4 & -8 & -12 \\ 0 & 8 & 16 \end{array}\right) \sim \left(\begin{array}{cc|c} -4 & 0 & 4 \\ 0 & 8 & 16 \end{array}\right) \\ &\Rightarrow \begin{cases} -4x + 0y = 4 & (1') \\ 0x + 8y = 16 & (2') \end{cases} \end{aligned}$$

This is Gaussian elimination

For completeness: A formal definition.

A system of linear equations is of the form

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\
&\vdots \\
a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m,
\end{aligned}
$$

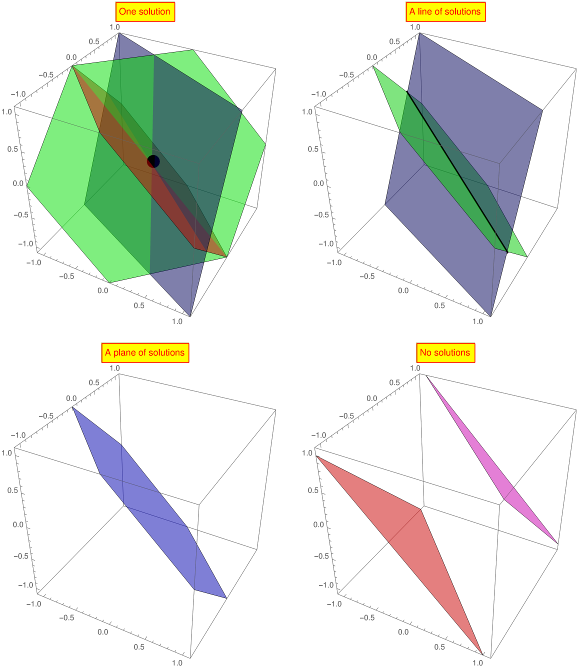
where x_i are variables and $a_{ij} \in \mathbb{K}$

Equivalently, one wants to solve

$$
\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \ddots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}
$$

for the variables x_i

All that can happen is...



Thank you for your attention!

I hope that was of some help.