What is...a quotient vector space?

Or: Identifying information.

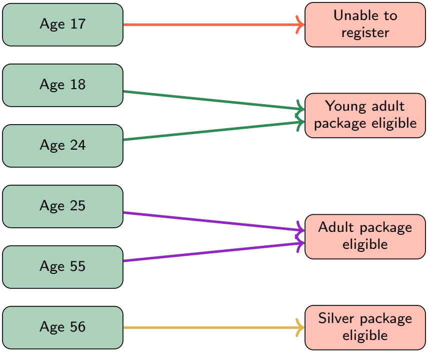
The idea of quotients.

V is a type XYZ object, W some subobject. Then a quotient V/W should satisfy:

- V/W should be of type XYZ
- The information in W should be trivial in V/W
- Information in V/W is equal if and only if it differs by W

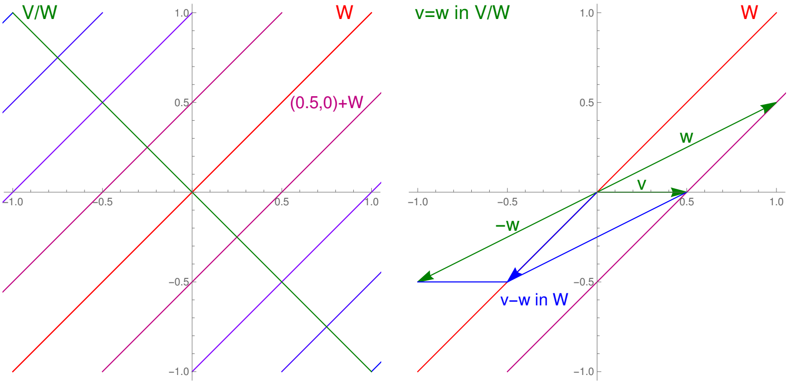
A quotient identifies information

In some sense things are the same:



Linear identification along codim 1

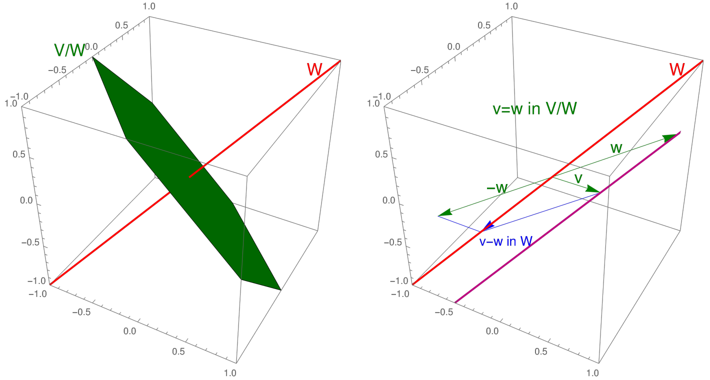
What happens if we collapse a line $W = \mathbb{R}(1,1)$ in $V = \mathbb{R}^2$ to a point?



The lines parallel to W are the points of V/W, $\dim V/W = 1$

Linear identification along codim 2

What happens if we collapse a line $W = \mathbb{R}(1,1,1)$ in $V = \mathbb{R}^3$ to a point?



The lines parallel to W are the points of V/W, $\dim V/W = 2$

For completeness: A formal definition.

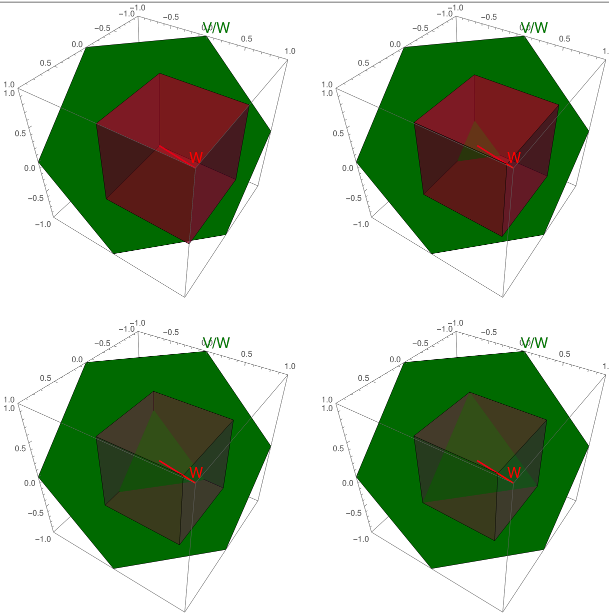
Let V be a vector space, W be a linear subspace. Define V/W by:

- Define an equivalence relation $\sim$ on V by stating that $v \sim w$ if $v - w \in W$
- $V/W = V/\sim$
- Scalar multiplication $\lambda[v] = [\lambda v]$ and addition $[v] + [w] = [v + w]$

Important facts about V/W:

- V/W is a vector space and $\dim V/W = \dim V - \dim W$ be careful with infinities
- $[w]$ for $w \in W$ is the zero in V/W
- $[v] = [w]$ if and only if $v - w \in W$

What about shapes under quotients?



A square becomes a triangle (in some sense)

Thank you for your attention!

I hope that was of some help.