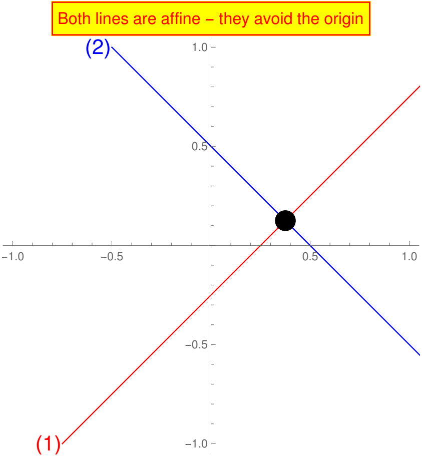
What is...an affine space?

Or: I lost my origin.

The intersection of affine lines

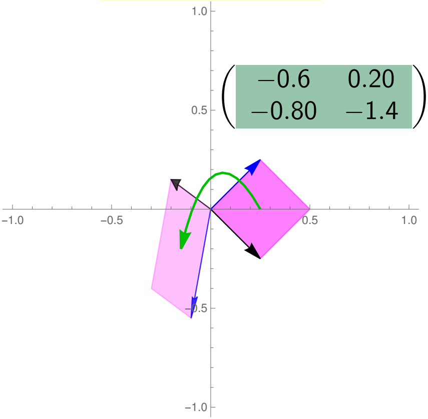
$$\begin{cases} x - y = 1/4 & (1) \\ x + y = -1/2 & (2) \end{cases}$$



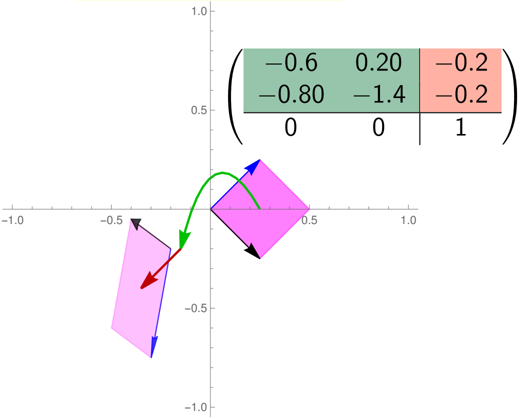
Affine spaces are the ingredients for systems of linear equations

Affine maps translate

A linear map keeps the origin

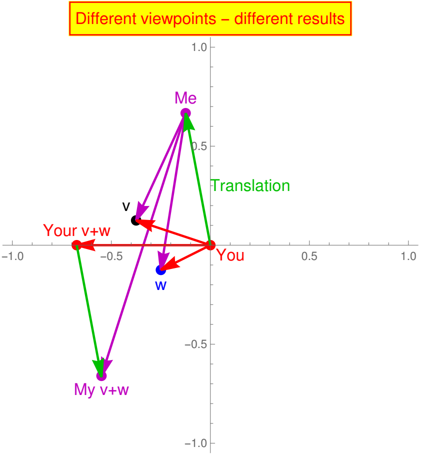


An affine map might move the origin



Affine maps “=” linear maps plus translation

Different origins



Different perspectives are related by translation

For completeness: A formal definition.

An affine space A over a field $\mathbb{K}$ is a set together with a vector space V, and a free, transitive action of the additive group of V on A. Explicitly, there exists a map

$$+ \colon A \times V \to A, (a, v) \mapsto a + v$$

such that:

(a) $a + 0 = 0$ Identity

(b) $(a + v) + w = a + (v + w)$ Associativity

(c) The map $v \mapsto a + v$ is a bijection $V \to A$ for all $a \in A$ free, transitive

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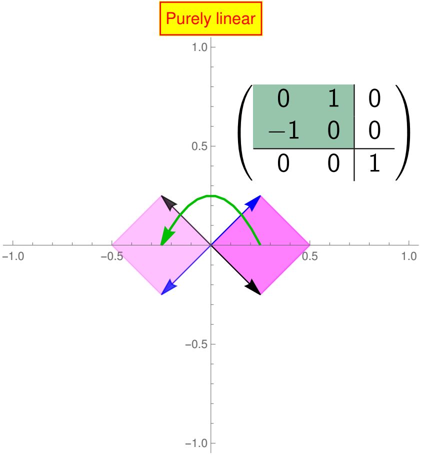
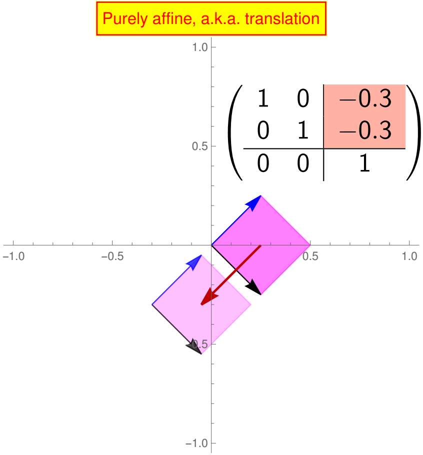
Affine maps are the the correct notion of maps between affine spaces:

$$\text{affine map} \colon f(a + v) = f(a) + f(v)$$

Matrices for affine maps





The point of this notation is that composition is matrix multiplication

Thank you for your attention!

I hope that was of some help.