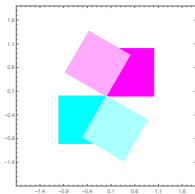


What is...matrix multiplication?

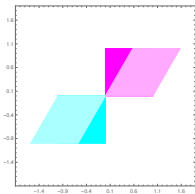
Or: Why the definition is not random.

Operations visually

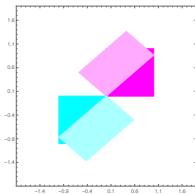
60 rotation matrix $\begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix}$



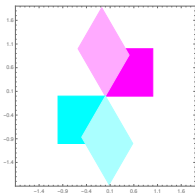
30 shearing matrix $\begin{pmatrix} 1 & 1/\sqrt{3} \\ 0 & 1 \end{pmatrix}$



First rotate, then shear $\begin{pmatrix} 1/2 & -1/\sqrt{3} \\ \sqrt{3}/2 & 1 \end{pmatrix}$



First shear, then rotate $\begin{pmatrix} 1 & -1/\sqrt{3} \\ \sqrt{3}/2 & 1/2 \end{pmatrix}$



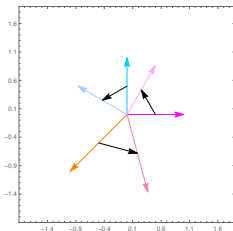
Wish. Matrix multiplication should be composition of actions!

Action on vectors

Where does $(1, 0)$ go? $\begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1/2 \\ \sqrt{3}/2 \end{pmatrix}$

Where does $(0, 1)$ go? $\begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -\sqrt{3}/2 \\ 1/2 \end{pmatrix}$

Where does $(-1, -1)$ go? $\begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix} \begin{pmatrix} -1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1/2 + \sqrt{3}/2 \\ -1/2 - \sqrt{3}/2 \end{pmatrix}$



Define matrix times vector as

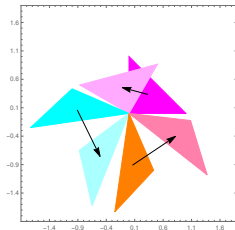
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = x \begin{pmatrix} a \\ c \end{pmatrix} + y \begin{pmatrix} b \\ d \end{pmatrix} = \begin{pmatrix} xa+yb \\ xc+yd \end{pmatrix}$$

Action on triangles

Where does the unit square go? $\begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix}$

Where does the cyan triangle go? $\begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix} \begin{pmatrix} -\sqrt{3} & -1/4 \\ -1 & \sqrt{3}/4 \end{pmatrix} = \begin{pmatrix} 0 & -1/2 \\ 2 & 0 \end{pmatrix}$

Where does the orange triangle go? $\begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix} \begin{pmatrix} -1/4 & \sqrt{3}/4 \\ -\sqrt{3} & -1 \end{pmatrix} = \begin{pmatrix} 11/8 & 5\sqrt{3}/8 \\ -5\sqrt{3}/8 & -1/8 \end{pmatrix}$

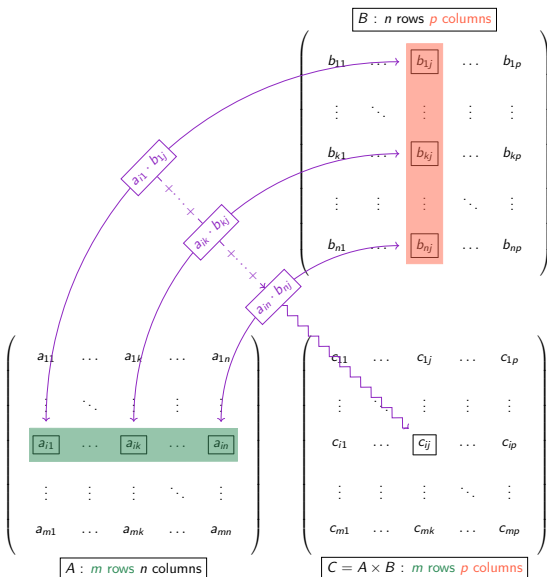


Define matrix times matrix as

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{pmatrix}$$

For completeness: A formal definition.

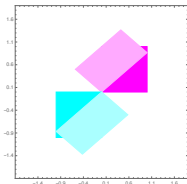
The matrix multiplication of a $m \times n$ matrix A and an $n \times p$ matrix B is defined as:



Life is not commutative.

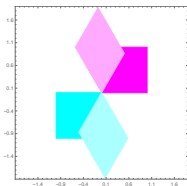
First rotate, then shear:

$$\begin{pmatrix} 1 & 1/\sqrt{3} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix} = \begin{pmatrix} 1/2 & -1/\sqrt{3} \\ \sqrt{3}/2 & 1 \end{pmatrix}$$



First shear, then rotate:

$$\begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix} \begin{pmatrix} 1 & \sqrt{3}/2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1/\sqrt{3} \\ \sqrt{3}/2 & 1/2 \end{pmatrix}$$



Thank you for your attention!

I hope that was of some help.