What is...machine learning in mathematics - part 10?

Or: Pattern detection

Latin squares

- Latin square = an n-by-n array filled with n different symbols
- Rule Each symbols occurs exactly once in each row and column
- Symbols = colors, to make it more colorful

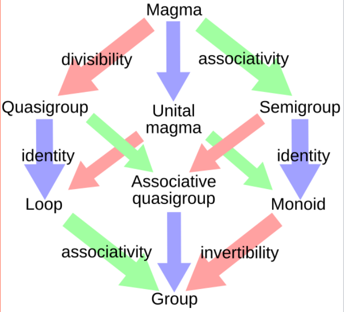
Cayley tables = multiplication tables of groups

$\cdot$	1	x	y	z
1	1	x	y	z
x	x	y	z	1
y	y	z	1	x
z	z	1	x	y

$\cdot$	1	x	y	z
1	1	x	y	z
x	x	1	z	y
y	y	z	1	x
z	z	y	x	1

- Cayley table = an *n*-by-*n* array filled with *n* different colors
- Rule Each color occurs exactly once in each row and column
- Additional rule = the composition needs to be associative

Groups $\neq$ Latin squares



Algebraic structures between magmas and groups: A *quasigroup* is a magma with the type of divisibility given by the Latin square property. A loop is a *quasigroup* with an identity element.

- Latin squares correspond to quasigroups
- Forgetting the unit, the additional rule encodes associativity
- Groups are thus a subset of Latin squares

Enter, the theorem

A (plain) neural network (NN) detected

groups among Latin squares with probability ≈90%

They run this for $n = 8$ and $n = 12$

- The NN thus "eyeballed associativity" – remarkable

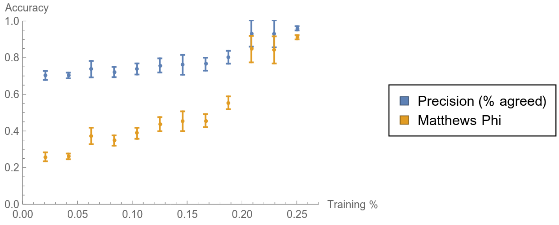
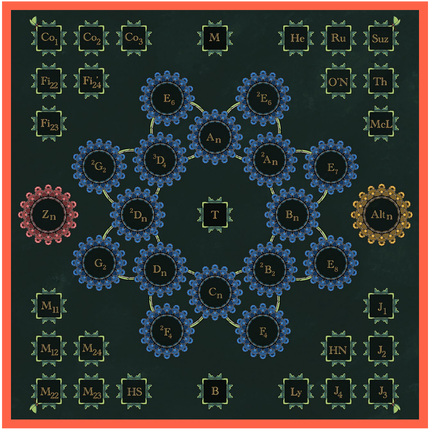


Figure 1: The learning curve for $n = 8$ Latin squares. We training a percentage of Latin squares, some of which are valid Cayley tables. This is then validated against the remaining unseen tables. The accuracy is measured by (1) precision and (2) the Matthews correlation coefficient ϕ, both of which already tend to 1 at about 1/4 of the data for training.

- Crucial The data set is biased (there are way more quasigroups than groups) and they needed to adjust for that

Here is another example ☺



- Above The classification of finite simple groups ("elements of group theory")
- This classification is one of the most remarkable theorems of the 20th century
- Fun Another (similar) NN was able to detect simple groups from Cayley tables with probability ≈90%

Thank you for your attention!

I hope that was of some help.