

What is...machine learning in mathematics - part 11?

Or: How to ask good questions!?

Questions versus answers



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- ▶ 42 = Answer to the ultimate question of life (but we forgot the question)
 - ▶ Mathematics is the art of asking questions
 - ▶ Goal (dream?) = let AI ask meaningful questions

Bottom-up and top-down

What is the most well-known math paper?

That's a great question—and while "most well-known" can depend on context (field of study, era, or audience), here are a few mathematical papers that are often considered iconic or especially influential across the board:

Most Well-Known/Influential Math Papers (General Consensus)

1. Euclid's *Elements* (circa 300 BC)

- Technically a book, not a paper—but it's the OG math text.
- Laid the foundation for geometry for over 2,000 years.
- Influenced everything from logic to the structure of mathematical proofs.

2. Andrew Wiles' proof of Fermat's Last Theorem (1995)

- Title: *Modular Elliptic Curves and Fermat's Last Theorem*
- Published in *Annals of Mathematics*
- Solved a 350-year-old problem—major media coverage, even in popular press.

3. Kurt Gödel's Incompleteness Theorems (1931)

- Title: *Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I*
- Showed that in any consistent formal system powerful enough to contain arithmetic, there are true statements that cannot be proven within the system.
- Fundamentally altered mathematical logic and philosophy.

► Math literature (books, papers,...) is (almost entirely) bottom-up

► Math research is (almost entirely) top-down

► AI is great for bottom-up (e.g. formalization) but top-down needs good questions

Automated conjecturing

Not just graph theory

C.E. Larson, N. Van Cleemput / Artificial Intelligence 231 (2016) 17–38

Table 2

Upper bound conjectures for the determinant of a symmetric matrix.

1.	$\det(x)$	\leq	$\text{permanent}(x)$
2.	$\det(x)$	\leq	$\text{minimum_eigenvalue}(x) \cdot \text{trace}(x)$
3.	$\det(x)$	\leq	$\text{maximum_eigenvalue}(x) \cdot \text{trace}(x)$
4.	$\det(x)$	\leq	$(\text{rank}(x) + 1)^{\text{spectral_radius}(x)}$
5.	$\det(x)$	\leq	$\text{permanent}(x) + \text{max_column_sum}(x) + 1$
6.	$\det(x)$	\leq	$\text{maximum}(\text{rank}(x), \text{minimum_eigenvalue}(x)^2)$
7.	$\det(x)$	\leq	$\text{maximum_eigenvalue}(x) \cdot \text{minimum}(\text{minimum_eigenvalue}(x), \text{trace}(x) + 1)$
8.	$\det(x)$	\leq	$\text{minimum_eigenvalue}(x) \cdot \text{minimum}(\text{trace}(x), \text{maximum_eigenvalue}(x))$
9.	$\det(x)$	\leq	$\text{maximum_eigenvalue}(x)^{\text{l_inf_norm}(x)} + \text{separator}(x)$
10.	$\det(x)$	\leq	$\text{trace}(x)^{\text{average_eigenvalue}(x)} - \text{permanent}(x)$
11.	$\det(x)$	\leq	$(\text{maximum_eigenvalue}(x) + 1)^{\text{minimum_eigenvalue}(x) \cdot \text{frobenius_norm}(x)}$

Table 3

Lower bound conjectures for the determinant of a symmetric matrix.

1.	$\det(x)$	\geq	$\text{minimum_eigenvalue}(x)^{\text{separator}(x)}$
2.	$\det(x)$	\geq	$\text{minimum}(\text{permanent}(x), \text{log}(\text{nullity}(x)))$
3.	$\det(x)$	\geq	$-2^{\text{l_inf_norm}(x) \cdot \text{rows}(x)} + \text{permanent}(x)$
4.	$\det(x)$	\geq	$-(\text{separator}(x) - 1)^{\text{frobenius_norm}(x)} + \text{permanent}(x)$
5.	$\det(x)$	\geq	$-1_{\text{inf_norm}(x)^{\text{frobenius_norm}(x)}}$
6.	$\det(x)$	\geq	$\text{minimum}(\text{rank}(x) - 1, \text{minimum_eigenvalue}(x))^{\text{nullity}(x)}$
7.	$\det(x)$	\geq	$-4^{\text{l_inf_norm}(x)^2} + \text{permanent}(x)$

- ▶ The same strategy has been applied in many fields
- ▶ Example above Conjectures about matrices
- ▶ Missing This method gives also many 'boring' conjectures – its a bit 'test all' instead fo something smarter – unclear how to fix this in 2024

- ▶ Automated conjecturing = no humans, please!
- ▶ Problem Most conjectures obtained this way are “boring”
- ▶ In 2025 we still need a fix for this

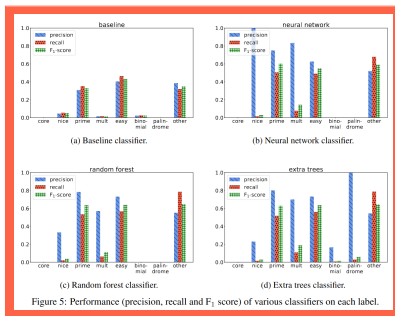
Enter, the theorem

A (plain) neural network (NN) detected

labels for integer sequences with $\approx 50\%$ accuracy

Quote: “[The results of this paper] seem to indicate that we can differentiate mathematically interesting OEIS sequences from random sequences [using AI]”

► Labels means things like “nice” :



► OEIS = (the most important?) online encyclopedia of integer sequences

Many sequences followed Benford's or Taylor's law!?

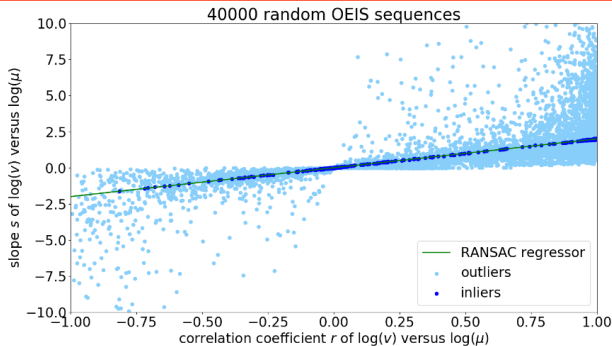


Figure 3: Slope s vs correlation coefficient r . The inliers (about half of the sequences) matches the regressor line with slope 2.001, intercept = 0.003 with a correlation coefficient equal to 0.999.

- ▶ **Example (Taylor's law)** The mean μ and the variance v in species data appear to satisfy a power law $v = T_a \mu^{T_b} \Rightarrow$ take log \Rightarrow get slope s , intercept b and correlation coefficient r
- ▶ **Quote** An interesting open question to investigate is why so many OEIS sequences follows $s = 2r$ so closely

Thank you for your attention!

I hope that was of some help.