What is...machine learning in mathematics - part 5?

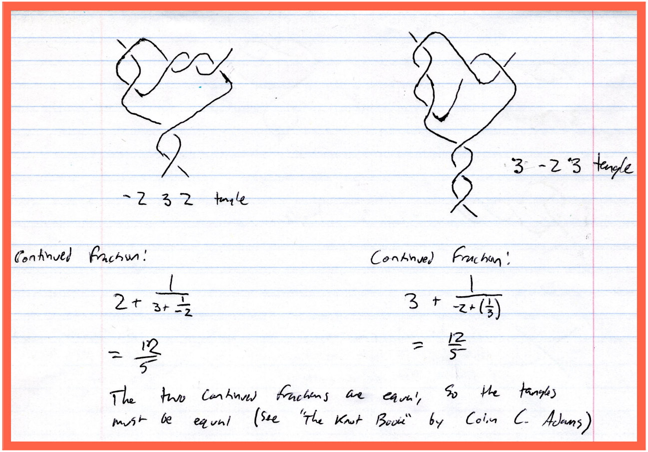
Or: Formula guessing

People like continued fractions

$$\varphi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ddots}}}}$$

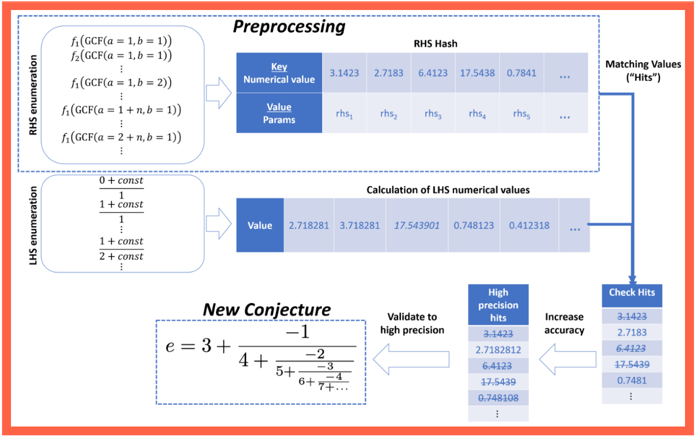
- Continued fraction $=$ an expression as above
- Every $x \in \mathbb{R}$ has a continued fraction expression, e.g. $\pi \approx 3 + 1/7 = 22/7$
- Fact People love them: there are many continued fraction expressions

They really like them



- **Fact** Every $x \in \mathbb{R}$ has multiple such expressions
- **Question** How to find such formulas?
- **Question - refined** How to find efficient formulas?

A simple algorithm



- Ramanujan machine = two algorithms based on matching numerical values
- It conjectures formulas without requiring any prior knowledge on any underlying mathematical structure
- Idea Computer utilizing numerical data to unveil mathematical structures

Enter, the theorem

The Ramanujan machine automated conjectures formulas for “famous” constants e.g.

$$1 + \pi/2 = 3 - 2/(6 - \ldots)$$

They tested the conjectures for up to 2000 digits of accuracy; several are proven

- Great This is an open source project and interacts with the community
- Here is a list of conjectured formulas

Novelty	Formula	Polynomials	Convergence $[\frac{digits}{term}]$
new and proven	$\frac{1+e}{-1+e} = 2 + \cfrac{1}{6+\cfrac{1}{10+\cfrac{1}{14+\cfrac{1}{\ddots}}}}$	$a_n = 4n+2,\ b_n = 1$	5.4905 *
new and proven	$\frac{3}{3-e} = 11 - \cfrac{10}{29-\cfrac{28}{55-\cfrac{54}{89-\cfrac{88}{\ddots}}}}$	$a_n = 2n(2n+7)+11,\ b_n = -2n(2n+3)$	4.9048 *
new and proven	$1 + \frac{e}{e-2} = 5 - \cfrac{4}{19-\cfrac{18}{41-\cfrac{40}{71-\cfrac{70}{\ddots}}}}$	$a_n = 2n(2n+5)+5,\ b_n = -2n(2n+1)+2$	4.9018 *
new and proven	$\frac{e}{-24+9e} = 6 - \cfrac{1}{7-\cfrac{2}{8-\cfrac{3}{9-\cfrac{4}{\ddots}}}}$	$a_n = 6+n,\ b_n = -n$	2.1756 *
new and proven	$\frac{e}{6-2e} = 5 - \cfrac{1}{6-\cfrac{2}{7-\cfrac{3}{8-\cfrac{4}{\ddots}}}}$	$a_n = 5+n,\ b_n = -n$	2.1698 *
new and proven	$\frac{1}{-16+6e} = 3 + \cfrac{1}{4+\cfrac{2}{5+\cfrac{3}{6+\cfrac{4}{\ddots}}}}$	$a_n = 3+n,\ b_n = n$	2.1695 *
new and unproven	$\frac{6e}{-3+2e} = 7 - \cfrac{4}{14-\cfrac{20}{23-\cfrac{54}{34-\cfrac{112}{\ddots}}}}$	$a_n = n(n+6)+7,\ b_n = -(n+3)n^2$	2.164 *
new and proven	$\frac{e}{-2+e} = 4 - \cfrac{1}{5-\cfrac{2}{6-\cfrac{3}{7-\cfrac{4}{\ddots}}}}$	$a_n = 4+n,\ b_n = -n$	2.164 *
new and proven	$\frac{1}{-5+2e} = 2 + \cfrac{1}{3+\cfrac{2}{4+\cfrac{3}{5+\cfrac{4}{\ddots}}}}$	$a_n = 2+n,\ b_n = n$	2.1638*
new and unproven	$\frac{3}{-10+4e} = 3 + \cfrac{4}{8+\cfrac{20}{15+\cfrac{54}{24+\cfrac{112}{\ddots}}}}$	$a_n = (n+1)(n+3),\ b_n = -(n+3)n^2$	2.1638 *
new and proven	$e = 3 - \cfrac{1}{4-\cfrac{2}{5-\cfrac{3}{6-\cfrac{4}{\ddots}}}}$	$a_n = 3+n,\ b_n = -n$	2.1581*
new and proven	$\frac{1}{-2+e} = 1 + \cfrac{1}{2+\cfrac{2}{3+\cfrac{3}{4+\cfrac{4}{\ddots}}}}$	$a_n = 1+n,\ b_n = n$	2.158 *
known	$\frac{1}{-1+e} = \cfrac{1}{1+\cfrac{2}{2+\cfrac{3}{3+\cfrac{4}{\ddots}}}}$	$a_n = n,\ b_n = n$	2.1522 *
new and proven	$\frac{e}{-1+e} = 2 - \cfrac{1}{3-\cfrac{2}{4-\cfrac{3}{5-\cfrac{4}{\ddots}}}}$	$a_n = 2+n,\ b_n = -n$	2.1522 *
new and unproven	$\frac{4e}{-1+2e} = 3 - \cfrac{3}{7-\cfrac{16}{13-\cfrac{45}{21-\cfrac{96}{\ddots}}}}$	$a_n = n(n+3)+3,\ b_n = -(n+2)n^2$	2.1493 *

Not quite!

Human:

$$\int_0^\infty \frac{1+\frac{x^2}{(b+1)^2}}{1+\frac{x^2}{a^2}} \times \frac{1+\frac{x^2}{(b+2)^2}}{1+\frac{x^2}{(a+1)^2}} \times \cdots \, dx = \frac{\sqrt{\pi}}{2} \times \frac{\Gamma\left(a+\frac{1}{2}\right)\Gamma(b+1)\Gamma(b-a+1)}{\Gamma(a)\Gamma\left(b+\frac{1}{2}\right)\Gamma\left(b-a+\frac{1}{2}\right)}.$$

$$1-5\left(\frac{1}{2}\right)^3+9\left(\frac{1\times 3}{2\times 4}\right)^3-13\left(\frac{1\times 3\times 5}{2\times 4\times 6}\right)^3+\cdots=\frac{2}{\pi}$$

$$1+9\left(\frac{1}{4}\right)^4+17\left(\frac{1\times 5}{4\times 8}\right)^4+25\left(\frac{1\times 5\times 9}{4\times 8\times 12}\right)^4+\cdots=\frac{2\sqrt{2}}{\sqrt{\pi}\,\Gamma^2\left(\frac{3}{4}\right)}.$$

AI:

$$\frac{e}{e-2} = 4 - \cfrac{1}{5 - \cfrac{1}{6 - \cfrac{2}{7 - \cfrac{3}{8 - \ddots}}}}$$

- Catch The Ramanujan machine uses very simple reasoning
- Ramanujan's original formulas are much more impressive
- Next step Maybe some more sophisticated AI can produce better results

Thank you for your attention!

I hope that was of some help.