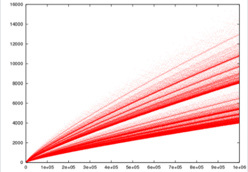
What is...machine learning in mathematics - part 7?

Or: Anti-conjecturing

Conjecturing is key



Number of ways to write an even number n as the sum of two primes (sequence A002375 in the OEIS)

- Conjecturing = the art of prediction
- Example (real-world) Market prediction is a form of conjecture
- Example (math) The Goldbach's conjecture ("sum of two primes")

Anti-conjecturing as well

What is an example of a famous conjecture in math that was disproved?

One famous example of a conjecture in mathematics that was disproved is **Euler's Sum of Powers Conjecture**. Proposed by Leonhard Euler in 1769, it generalized Fermat's Last Theorem. The conjecture stated that for any integer $n > 2$, at least n n-th powers are required to sum to another n-th power. In other words, it claimed that the equation:

$$a_1^n + a_2^n + \cdots + a_k^n = b^n$$

has no integer solutions if $k < n$.

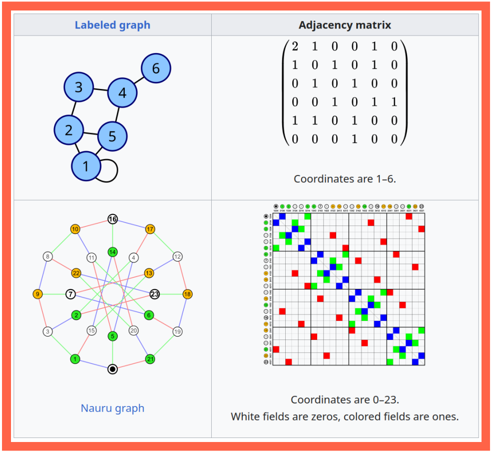
However, in 1966, this conjecture was disproved by Lander and Parkin, who found a counterexample for $n = 5$:

$$27^5 + 84^5 + 110^5 + 133^5 = 144^5.$$

This counterexample showed that only four 5th powers were needed to sum to another 5th power, disproving Euler's conjecture for $n = 5$. Later, further counterexamples were found for other values of n, solidifying the disproof.

- **Anti-conjecture** = disproving a conjecture (not standard terminology)
- **Example** Euler's sum of powers conjecture is wrong (*)
- **Observation** Conjecture (*) was wrong due to a "small number coincidence"

A conjecture in graph theory



- **Distance matrix** = matrix of distances; **Distance eigenvalues/spectrum** = its eigenvalues $\lambda_1 \geq ... \geq \lambda_n$; π=proximity and D=diameter, $d = \lfloor 2/3D \rfloor$
- **Conjecture** We have $\pi + \lambda_d > 0$ for a connected graph on $n \geq 4$ vertices

Enter, the theorem

Via reinforcement learning a neural network (NN) found

almost counterexamples with a clear pattern for $n = 4, \ldots, 30$

Then they constructed a counterexample, following the pattern, by hand ($n = 203$)

- Reinforcement learning = give points for good scores
- Here is the evolution of a similar pattern:

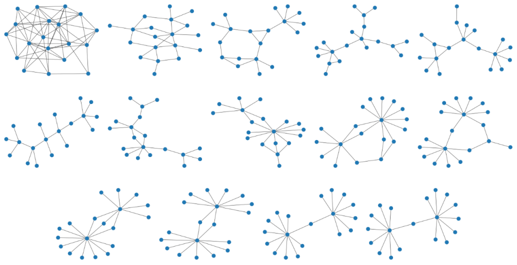
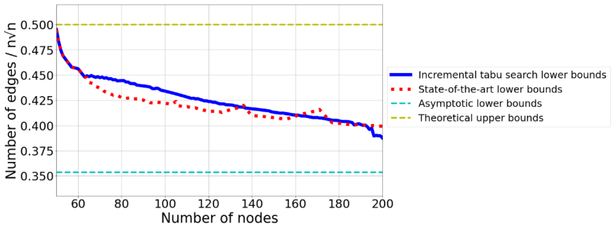


Figure 3: The evolution of the best construction over time. The network quickly realizes that sparse graphs are best, and eventually the "balanced double star" structure emerges.

This works well



$$\frac{1}{2\sqrt{2}} \leq \lim_{n\to\infty} \frac{f(n)}{n\sqrt{n}} \leq \frac{1}{2}.$$

- There are actually many other examples along the same lines
- Above Minimizing a certain value of a certain family of graphs
- Great Eventually the NN is better than by hand

Thank you for your attention!

I hope that was of some help.