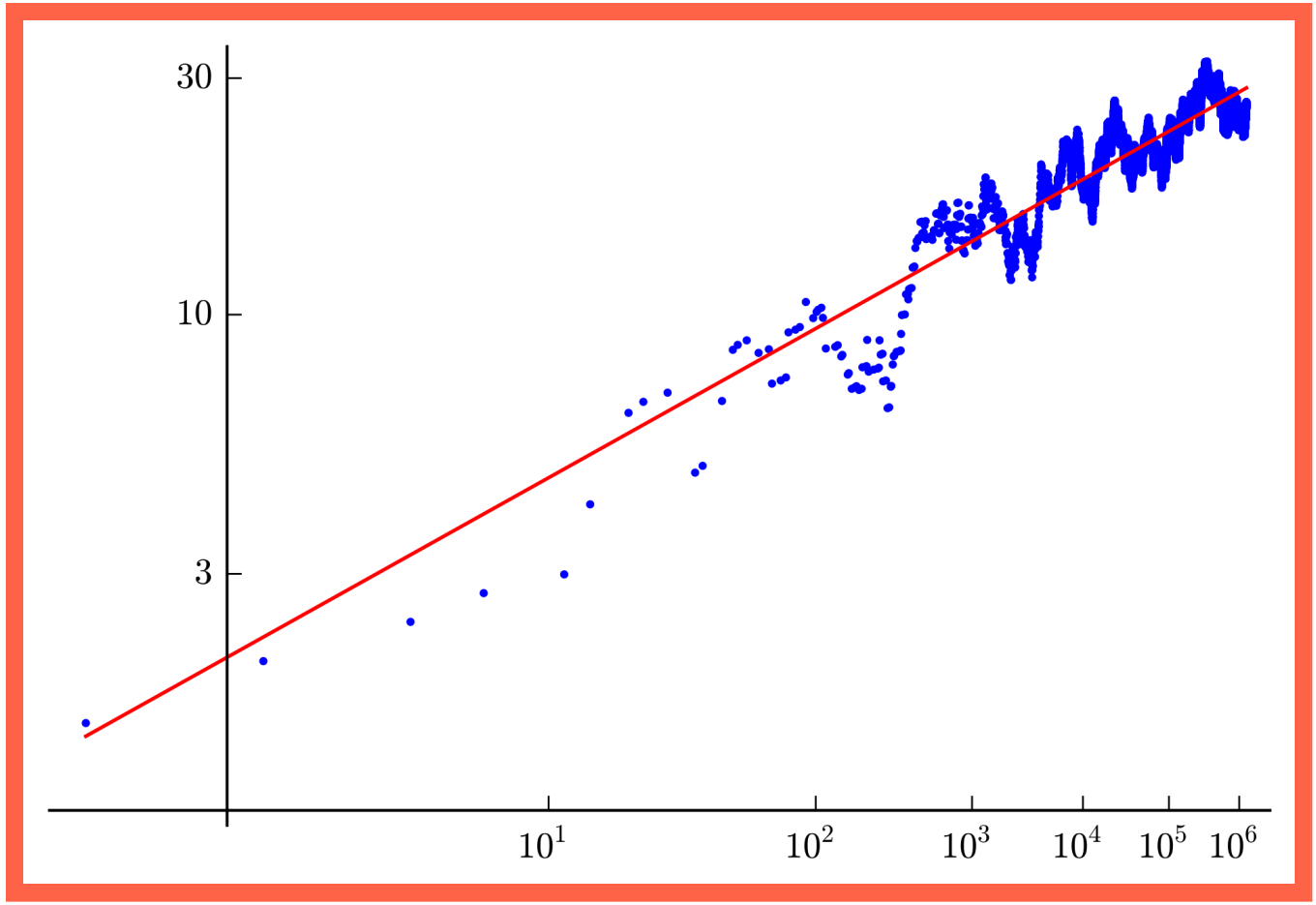
What is...machine learning in mathematics - part 9?

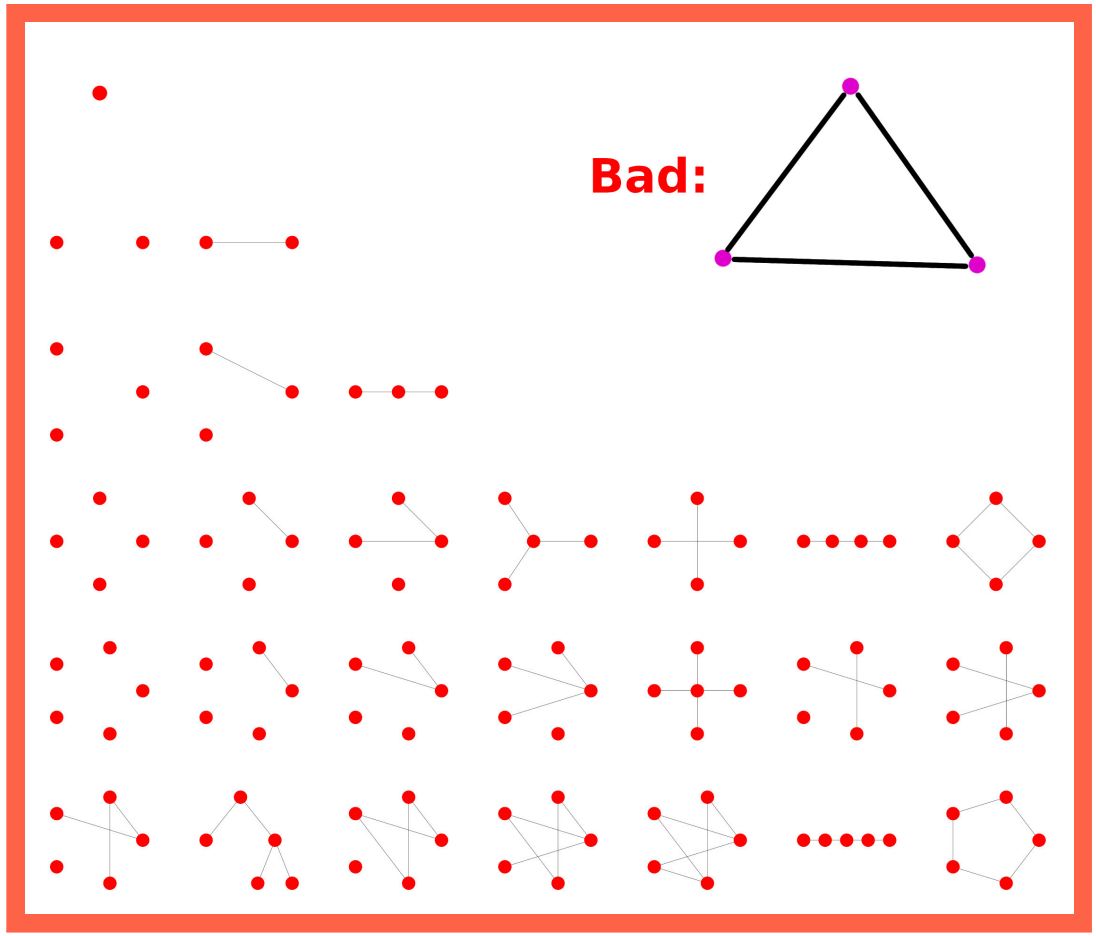
Or: Example creation

Examples, please!



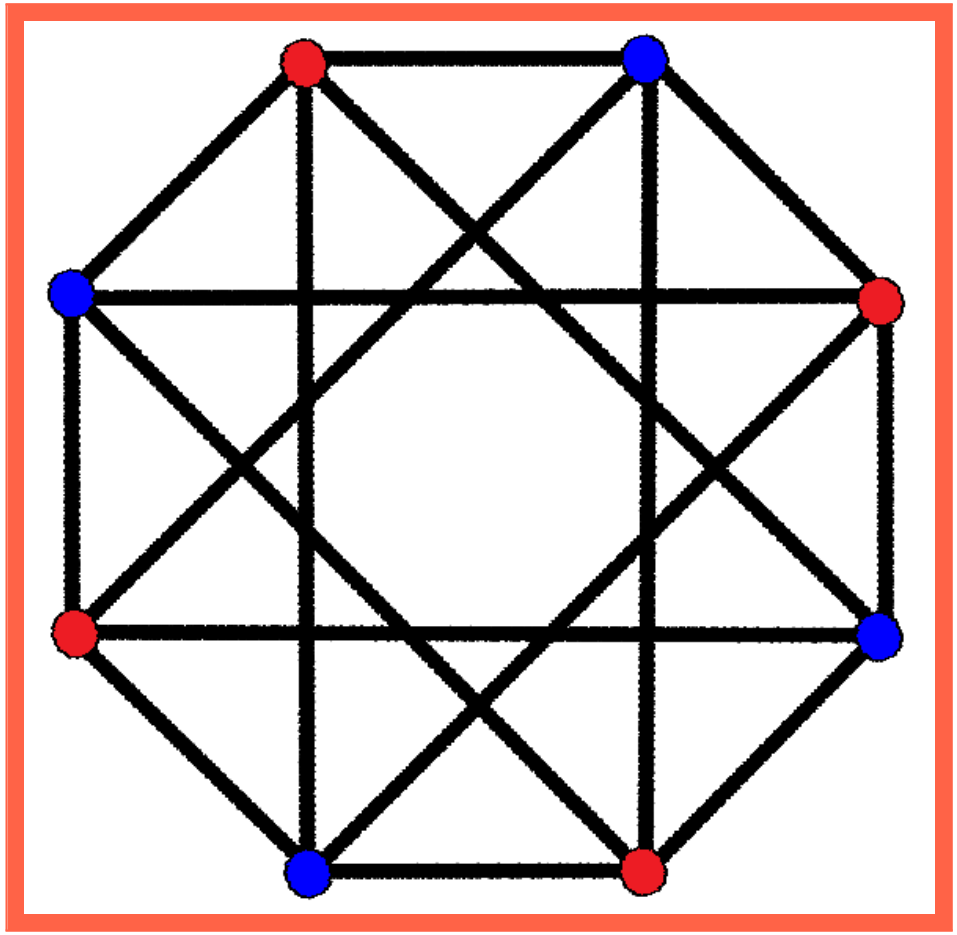
- Above The Birch–Swinnerton-Dyer conjecture was discovered via examples
- Examples are a crucial part of mathematics and life itself
- Idea AI should be able to generate examples!

Extremal graph theory



- Triangle = see above
- Triangle free = no three vertices form a triangle
- Question What is the maximal number of edges in a graph on n vertices with no triangles?

Examples needed!



- Mantel's theorem The maximal number of edges of a triangle free graph with n vertices is

$$\lfloor n^2/4 \rfloor$$

- The extremal graph in this case the complete bipartite graph $K_{n/2,n/2}$ (above)

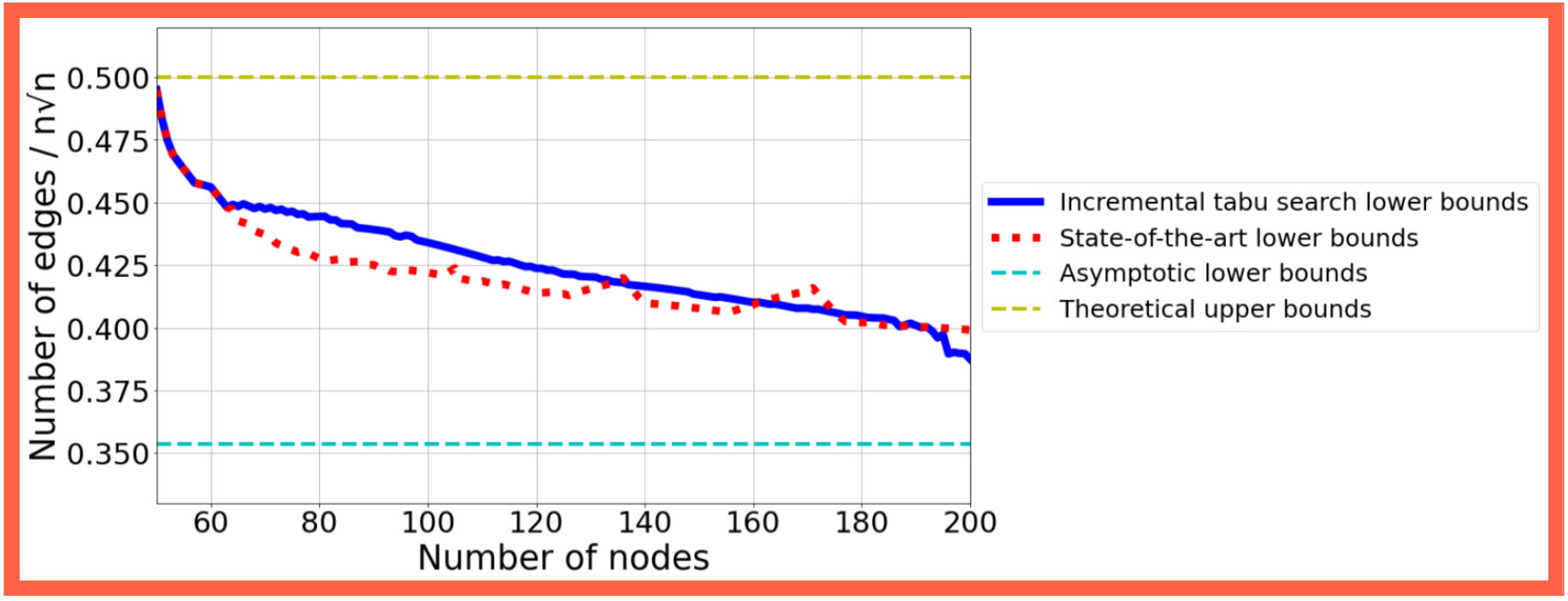
Enter, the theorem

A neural network (NN) found (nearly) extremal graphs

avoiding triangles (t) and squares (s) for $n = 60, \ldots, 200$ vertices

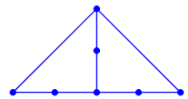
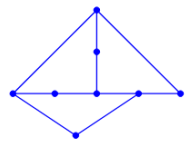
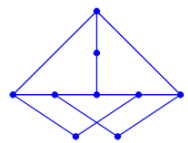
They also found $f(n)$ =maximal number of edges of a n vertex graph avoiding t and s

- They used a beefed-up version of a graph NN and reinforcement learning
- It was known that $\frac{1}{2\sqrt{2}} \leq \frac{f(n)}{n\sqrt{n}} \leq \frac{1}{2}$ and they improved the known bounds



Here are a few examples ☺

Table 1: The values of $f(n)$ for $n = 1, 2, \ldots, 9$.

n	$f(n)$	all n-node feasible graphs with $f(n)$ edges
1	0	
2	1	
3	2	
4	3	
5	5	
6	6	
7	8	
8	10	
9	12	

- Note to myself: show examples ☺
- Above Values for n, $f(n)$ and extremal graphs
- Observation There is some pattern but its not obvious – perfect for a NN

Thank you for your attention!

I hope that was of some help.