

What is...AI-assisted math?

Or: Can a machine help us formulate a theorem?

1956: Logic Theorist searches for proofs



Celebrating AAAI's 25th Anniversary

Herbert Simon (left) in a chess match against Allen Newell c. 1958



- ▶ The achievement Find proofs of theorems (à la Principia Mathematica)
- ▶ The idea Guide the search using mathematical rules of thumb
- ▶ The next question Could a machine also suggest what to prove?

1960s–1980s: mathematical knowledge as rules

FIGURE 1: Concepts Initially Given to AM

Below is a graph of the concepts which were present in AM at the beginning of its run. Single lines denote Generalization/Specialization links, and triple lines denote Examples/Is-a links.

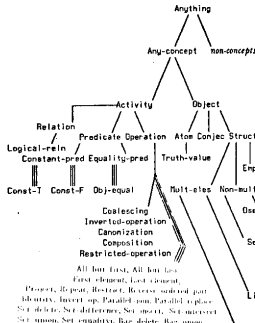


FIGURE 2: Concepts Discovered by AM

The list below is ordered by size, and the range of AM's exploration of it is, far from complete, only some of the concepts were explored by it. The concepts are listed in the order in which they were defined. In place of the usually arbitrary name chosen by AM, I make provision for the standard mathematical name for the concept, or else a brief description of what it is.

Sets with less than 2 elements (Singletons and empty sets)
 Sets with no atoms (atoms are units of interest)
 Bags containing a 1 copy of at most 1 kind of element
 Supersets containing
 Doubleton bags and sets
 Set membership
 Dependent bags
 Subset
 Disjoint sets
 Same length
 Same first element
 Equal length
 Numbers (in general)
 Add
 Minus
 $N \times M \times K \times L$
 Less than or equal to
 Times
 Composite F with divisors $F = P \times Q$
 Invert structure S into S^{-1}
 Try to delete structure S from S^{-1} to leave
 Double (Add $\times 2$ to itself)
 Subtract $\times 2$ from itself
 Square of $\times 2$ (Add $\times 2$)
 Double or square to $\times 8$ (Add $\times 8$)
 up 1, and repeat (Add $\times 8$ along S)
 Composite pairs, append together $F \times G$ for rank of S
 Double or square to place rank of S into S for S
 Composite three operations $\times 2 \times 2 \times 2$ (Add $\times 8$)
 Composite three operations $\times 2 \times 2 \times 2$ (Add $\times 8$)
 Add $\times 8$ all ways to represent the sum of non-zero numbers
 $G \times H$ set H (Add $\times 8$ is always defined whatever H is)
 Invert or Invert
 Size $\times$ Add $\times 1$ (Is $\times 1$) The number of ways to partition n
 Cubing
 Exponentiation
 Halfway (for natural numbers only) then Halfway (Is $\times 1$)

- A central idea Represent knowledge by symbols and explicit rules
- The ambition Automate the search for concepts and conjectures
- For example AM explores elementary number theory using heuristics

1990s–2000s: cooperating specialist tools

A Sample [Otter](#) Proof

Theorem: A [Robbins algebra](#) satisfying $x+x=x$ is a Boolean algebra.

Here is Otter's proof in [Son of BirdBrain](#)'s format:

Searching ...

Success, in 1.28 seconds!

```
----- PROOF -----
1      n(n(A)+B)+n(n(A)+n(B))!=A.      []
2      x=x.                             []
3      x+y=y+x.                         []
5,4    (x+y)+2=x+ (y+z).               []
6      n(n(x+y)+n(x+n(y)))=x.          []
8      x+x=x.                           []

18     n(n(A)+n(B))+n(n(A)+B)!=A.        [para from,3,1]
13     x+ (x+y)=x+y.                   [para_into,4,8,flip,1]
15     x+ (y+z)=y+ (x+z).               [para_into,4,3,demod,5]
23,22  x+ (y+x)=x+y.                   [para_into,13,3]
26     n(n(x)+n(x+n(x)))=x.            [para_into,6,8]
36     n(n(n(x)+x)+n(n(x)))=n(x).       [para_into,6,8]
42     n(n(x+n(y))+n(x+y))=x.          [para_into,6,3]
52     x+ (y+z)=x+ (z+y).              [para_into,15,3,demod,5]
81,80  n(n(x+n(x))+n(x))=x.            [para_into,26,3]
82     n(n(n(x)+x)+n(x))=n(x).          [para_from,26,6,demod,23]
125    n(n(n(x+n(x))+ (n(x)+x))+x)=n(x+n(x))+n(x). [para_into,80,80,demod,5,81]
139    n(n(n(x+n(x))+x)+x)=n(x+n(x)).   [para_from,80,6]
166,165 n(n(x+n(x))+x)=n(x).            [para_into,82,3]
180,179 n(n(x)+x)=n(x+n(x)).           [back_demod,139,demod,166]
195    n(n(x+n(x))+n(n(x)))=n(x).       [back_demod,36,demod,180]
197    n(n(x+ (n(x)+n(x+n(x))))+ (n(x+n(x))+x))=n(x). [para_into,165,165,demod,5,180,5,166]
206,205 n(n(x+ (n(x)+n(x+n(x))))+n(x))=n(x+n(x))+x. [para_from,165,80,demod,166,5,180,5]
223,222 n(n(x+y)+ (y+x))=n(x+ (y+n(x+y))). [para_into,179,52,demod,5]
231,230 n(n(x+ (n(x)+n(x+n(x))))+x)=n(x+n(x))+n(x). [back_demod,125,demod,223]
564,563 n(x+n(x))+x=x.                  [para_into,195,80,demod,5,223,81,206,81]
582,581 n(x+n(x))+n(x)=n(x).           [back_demod,197,demod,564,231]
586,585 n(n(x))=x.                     [back_demod,80,demod,582]
606,605 n(x+n(y))+n(x+y)=n(x).         [para_into,585,42,flip,1]
621    A=A.                             [back_demod,10,demod,606,586]
622    $F.                             [binary,621,2]

----- end of proof -----
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- The approach Combine conjecturing with automated reasoning
- The challenge Identify useful statements among the many candidates
- For example HR builds and tests theories about finite groups

From: Constructions in combinatorics via neural networks

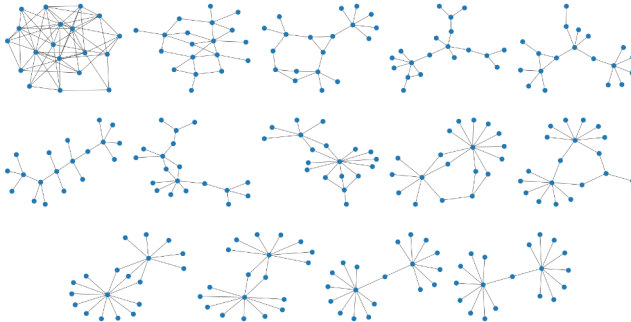
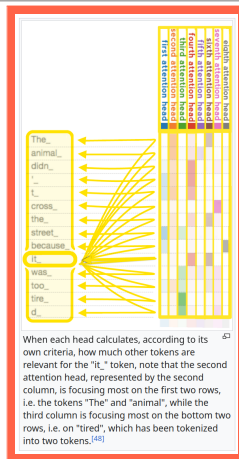


Figure 3: The evolution of the best construction over time. The network quickly realizes that sparse graphs are best, and eventually the “balanced double star” structure emerges.

- The toolkit Large datasets, neural networks and reinforcement learning
- The hope Learn which mathematical patterns and choices matter
- For example Neural networks help guide a new knot theory theorem

2023 onwards: language models in the research workflow



- ▶ **The ambition** Use one model across conjecturing, coding and proof attempts
- ▶ **The method** Connect its suggestions to computation and verification
- ▶ **For example** AlphaEvolve searches for new mathematical constructions

Thank you for your attention!

Next time: Condensed mathematics