# What is...extremal graph theory?

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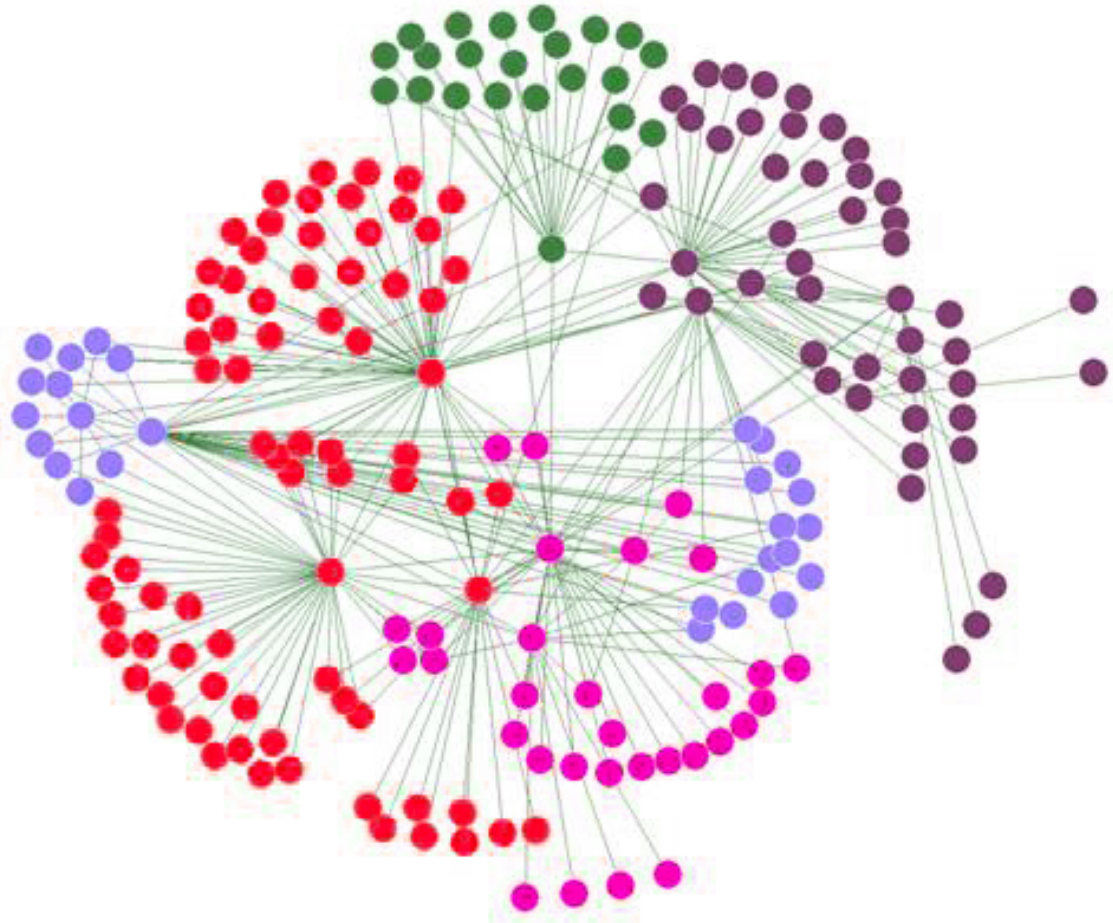
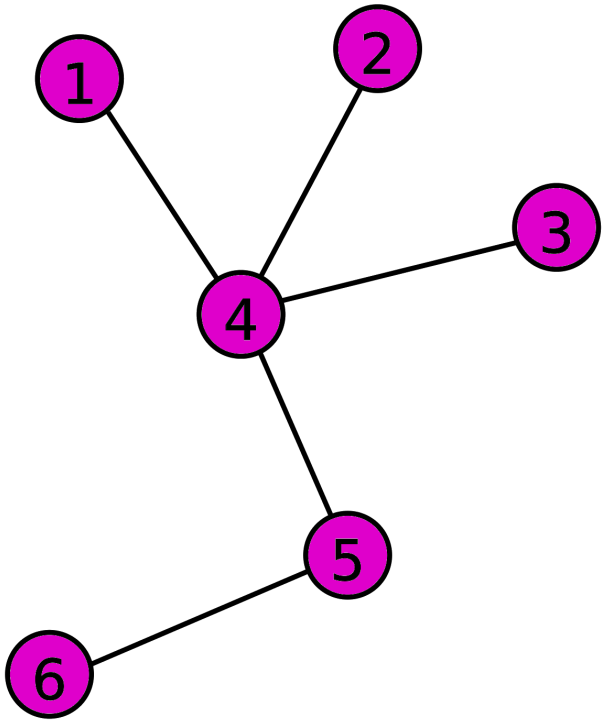
Or: Subfields of mathematics 1

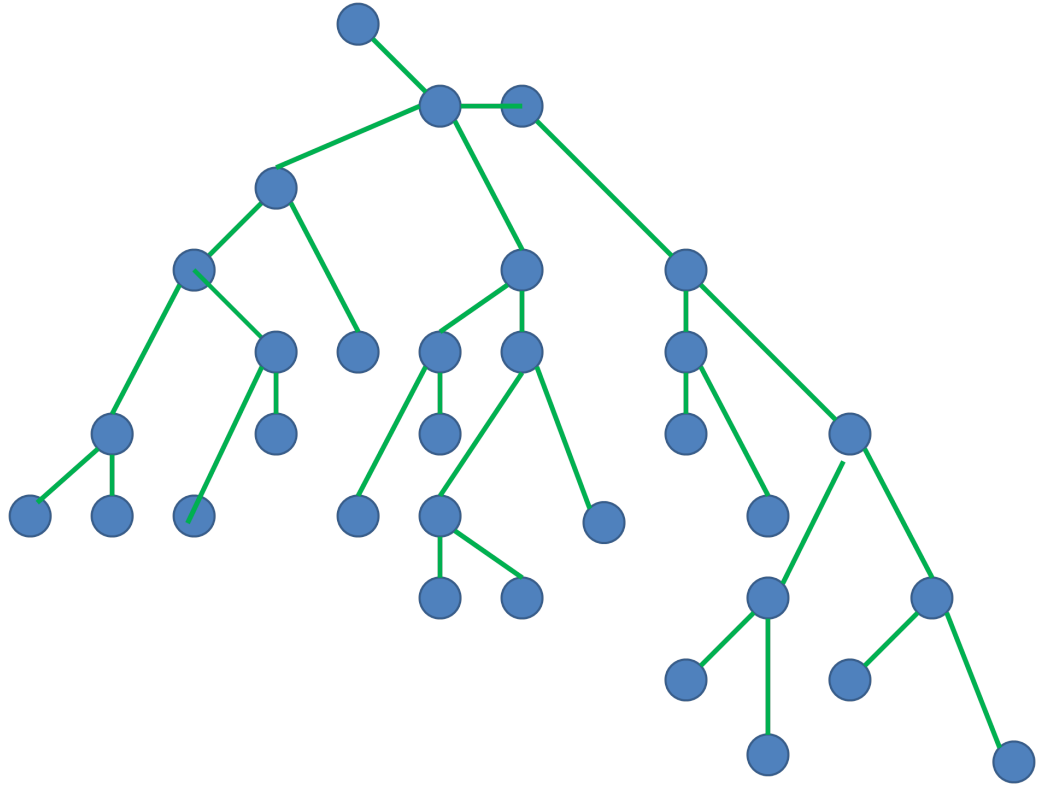
# Graph theory



- **Graph** (network outside of math) = vertices and edges
- **Example** Human relationships form graphs
- **Graph theory** = the study of graphs; very successful field of math

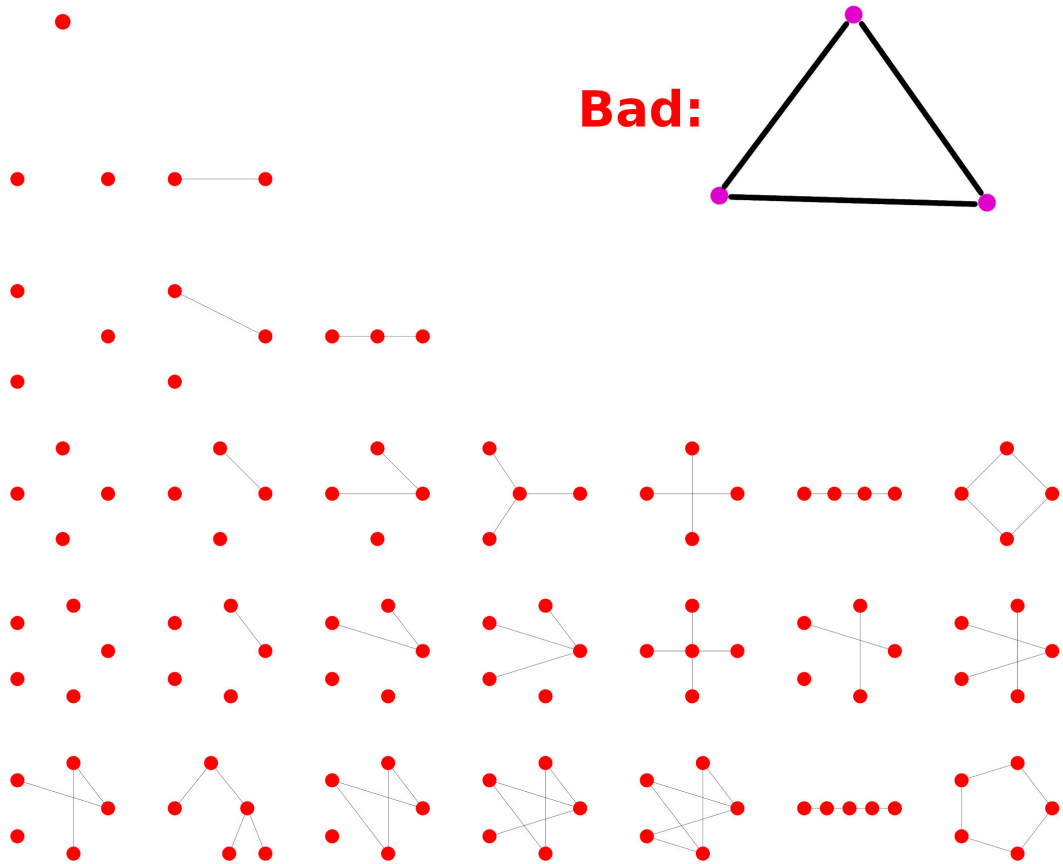
# Extremal questions



- Extremal graph theory tries to identify "extremal structures", e.g. "What is the maximal X under assumption Y?"
- Example question What is the maximum number of edges in a graph on $n$ vertices with no cycles?
- Example answer The class of graphs is trees; they have $n - 1$ edges

# Triangle free graphs




- Triangle = see above
- Triangle free = no three vertices form a triangle
- Question What is the maximal number of edges in a graph on $n$ vertices with no triangles?

## Enter, the theorem

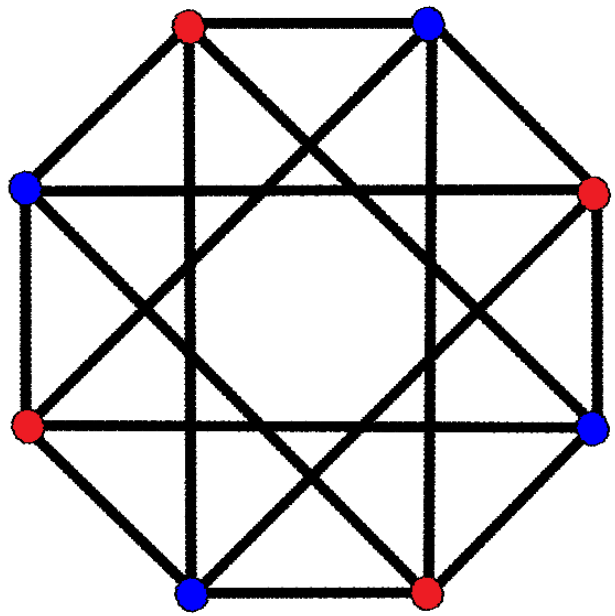
The maximal number of edges of a triangle free graph with $n$ vertices is

$$\lfloor n^2/4 \rfloor$$

This is Mantel's theorem

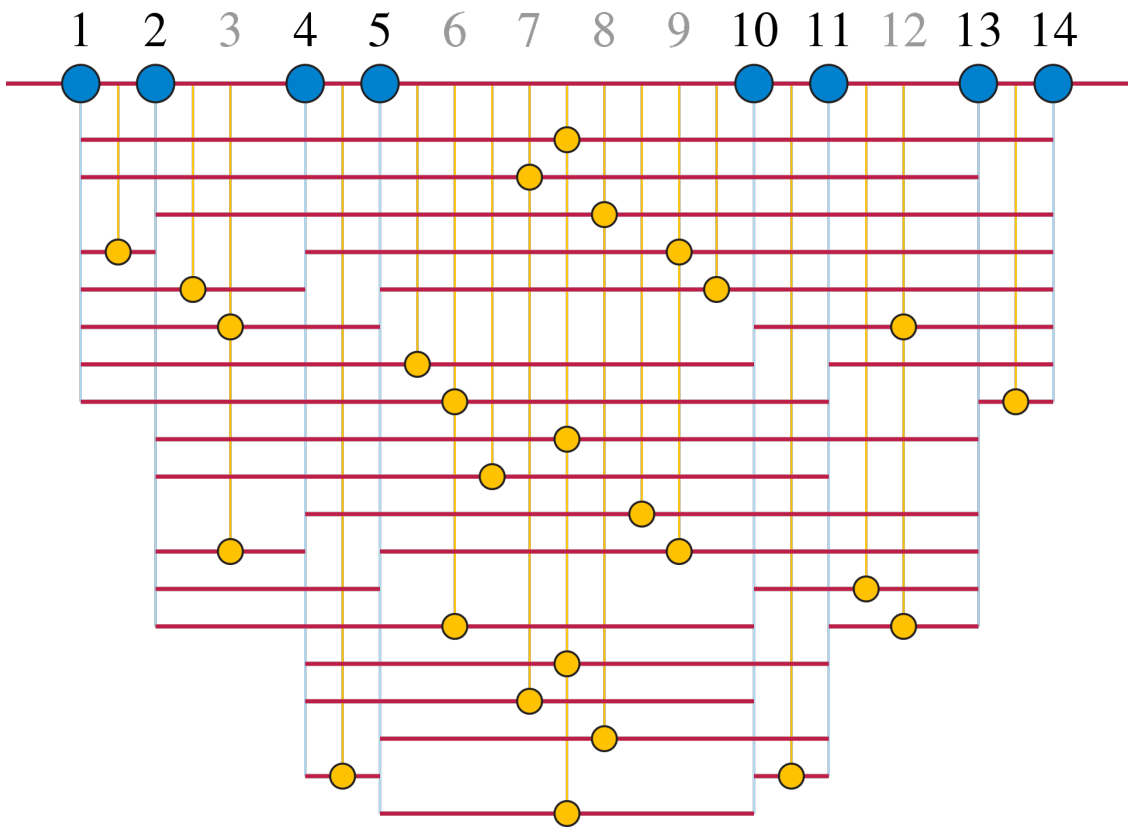
- The extremal graph in this case the complete bipartite graph $K_{n/2,n/2}$, e.g.:

$K_{4,4} =$



- Extremal graph theory answers similar questions!

# Roth's theorem




- Analog theorems are known throughout mathematics
- Example from number theory Roth's theorem
- Statement Every subset of $\mathbb{N}$ with density $> 0$ contains a 3-term arithmetic progression

**Thank you for your attention!**

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I hope that was of some help.