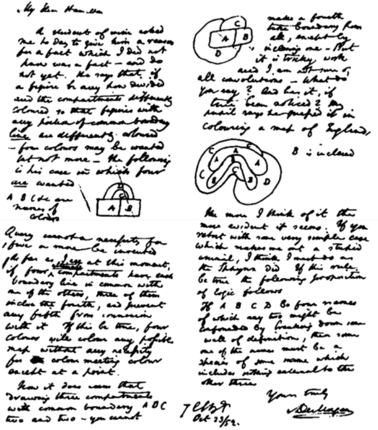
What is...computer aided mathematics?

Or: Subfields of mathematics 10

Four color theorem (4CT)



My dear Hamilton

A student of mine asked me to day to give him a reason for a fact which I did not know was a fact — and do not yet. He says that if a figure be any how divided and the compartments differently coloured so that figures with any portion of common boundary line are differently coloured — four colours may be wanted but not more — the following is his case in which four are wanted

A B C D are names of colours

Query cannot a necessity for five or more be invented? As far as I see at this moment, if four ultimate compartments have each boundary line in common with one of the others, three of them inclose the fourth, and prevent any fifth from connexion with it. If this be true, four colours will colour any possible map without any necessity for colour meeting colour except at a point.

Now it does seem that drawing three compartments with common boundary ABC two and two — you cannot make a fourth take boundary from all, except by inclosing one — But it is tricky work and I am not sure of all convolutions — What do you say? And has it, if true been noticed? My pupil says he guessed it in colouring a map of England.

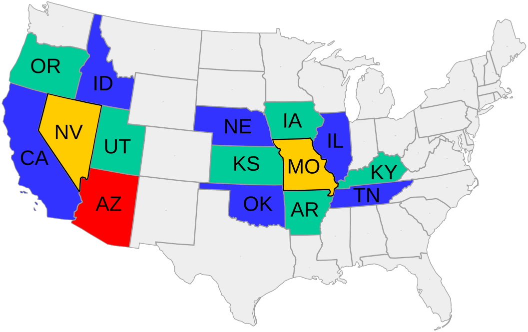
B is inclosed

The more I think of it the more evident it seems. If you retort with some very simple case which makes me out a stupid animal, I think I must do as the Sphynx did. If this rule be true the following proposition of logic follows

If A B C D be four names of which any two might be confounded by breaking down some wall of definition, then some one of the names must be a species of some name which includes nothing external to the other three.

Yours truly
ADeMorgan

Oct 23/52.



- Task Color countries such that two countries that share a border get different colors
- Above We might need four colors
- 4CT We need at most four colors

4CT – a first proof

Appel–Haken's proof ∼**1976** has
770ish pages
with 500 pages of
"cases to check"
The book is from ∼**1989**
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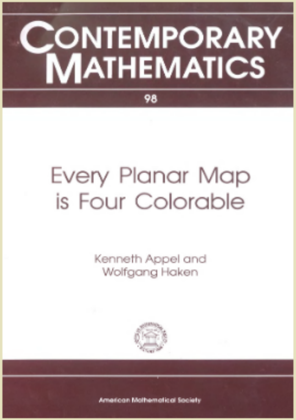
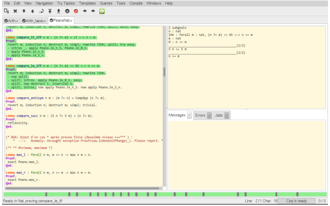


- The 4CT was the first major theorem with a computer assisted proof
- Problem There are many special cases one needs to check
- Important A computer was used as a helping hand not as the main character

Formal Proof—The Four-Color Theorem

Georges Gonthier



- Coq = interactive theorem prover first released in 1989
- In the early 2000s the 4CT was computer verified
- Important A computer was used to double check not as the main character

Enter, the theorem

Computer verification is everywhere

Topology

Topology and Metric Spaces topology of a metric space, induced topology, finite product of metric spaces, limits of sequences, cluster points, continuous functions, homeomorphisms, compactness in terms of open covers (Borel-Lebesgue), sequential compactness is equivalent to compactness (Bolzano-Weierstrass), connectedness, connected components, path connectedness, Lipschitz functions, uniformly continuous functions, Heine-Cantor theorem, complete metric spaces, contraction mapping theorem.

Normed vector spaces on $\mathbb{R}$ and $\mathbb{C}$ topology on a normed vector space, Banach open mapping theorem, equivalence of norms in finite dimension, norms $\|\cdot\|_p$ on $\mathbb{R}^n$ and $\mathbb{C}^n$, absolutely convergent series in Banach spaces, continuous linear maps, norm of a continuous linear map, uniform convergence norm (sup-norm), normed space of bounded continuous functions, completeness of the space of bounded continuous functions, Heine-Borel theorem (closed bounded subsets are compact in finite dimension), Riesz' lemma (unit-ball characterization of finite dimension), Arzela-Ascoli theorem.

Hilbert spaces Hilbert projection theorem, orthogonal projection onto closed vector subspaces, dual space, Riesz representation theorem, inner product space l^2, completeness of l^2, inner product space L^2, completeness of L^2, Hilbert bases, example, the Hilbert basis of trigonometric polynomials, Lax-Milgram theorem.

https://leanprover-community.github.io/undergrad.html keeps track of computer verification of undergrad math

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- The above is just one example out of many
- In particular, there are many computer verified proofs nowadays
- Computer aided mathematics answers similar questions!

Why should I learn proofs?

Home › Journal of Automated Reasoning › Article

Solution of the Robbins Problem

Published: December 1997

Volume 19, pages 263–276, (1997) Cite this article

Abstract

In this article we show that the three equations known as commutativity,associativity, and the Robbins equation are a basis for the variety ofBoolean algebras. The problem was posed by Herbert Robbins in the 1930s. Theproof was found automatically by EQP, a theorem-proving program forequational logic. We present the proof and the search strategies thatenabled the program to find the proof.

- Problem There are still humans involved
- Goal Computer automated proofs where the machine is the main character
- Sadly we are still in the early stages

Thank you for your attention!

I hope that was of some help.