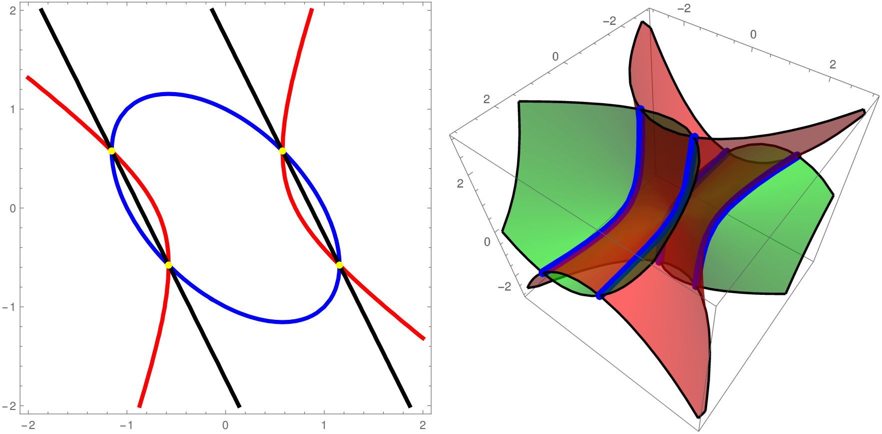
What is...tropical geometry?

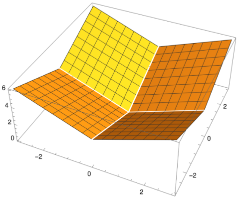
Or: Subfields of mathematics 11

Algebraic geometry (AG)



- AG = the study of zero sets of polynomials (with multiple variables)
- Since polynomials are everywhere AG is also everywhere
- Mild catch This is also very difficult

Piecewise linear world



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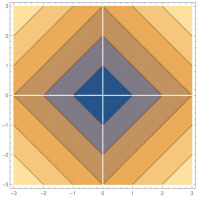

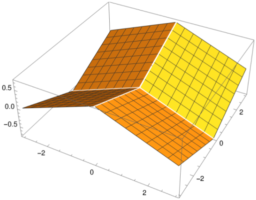

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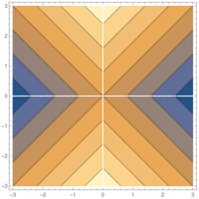
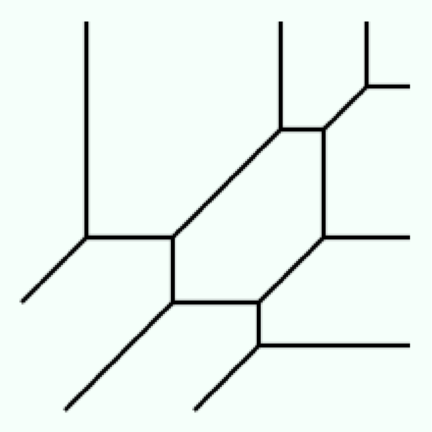
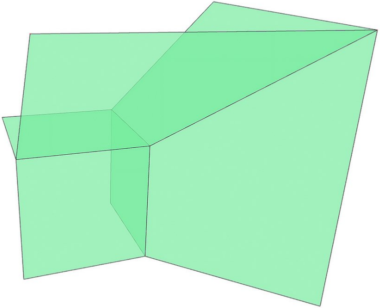


- The easiest case of AG are linear maps
- AG then generalizes this by considering higher degrees
- Alternative Why not make things piecewise linear instead?

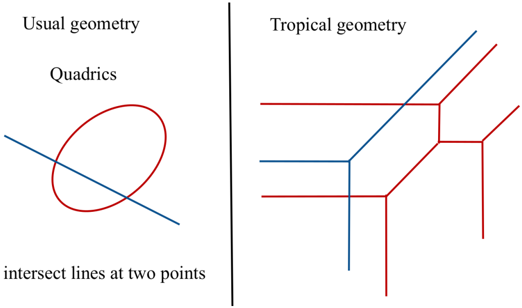
Tropical geometry (TG)

- TG = AG with piecewise linear "polynomials"
- The key is to change fields and then polynomials are piecewise linear
- Field for TG $\mathbb{R} \cup \{\infty\}$ with '$\oplus = \min$' and '$\otimes = +$' (a semiring not a field)

Enter, the theorem

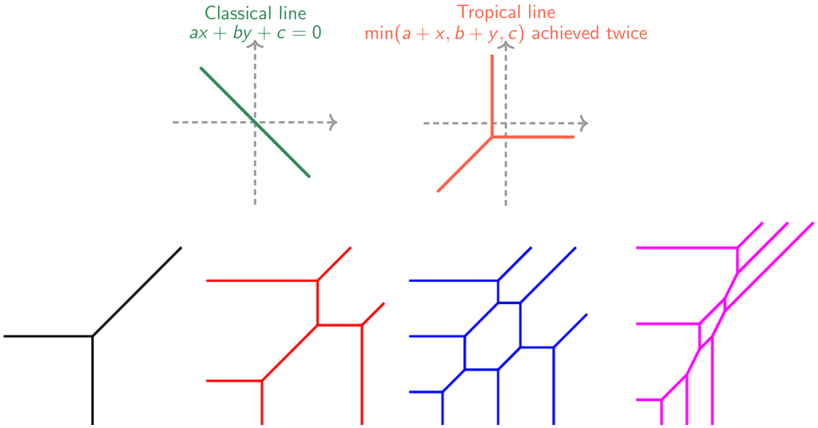
The tropical Bézout theorem holds



That is, # intersection points = the product of the degrees

- The above is just one example out of many where TG mimics AG
- Even better, TG can also be used to prove new theorems in AG
- Tropical geometry answers similar questions!

Some details on tropical stuff



- Tropical polynomial, for example:

$$(x \oplus y)^3 = (x \oplus y) \otimes (x \oplus y) \otimes (x \oplus y) = x^3 \oplus x^2 y \oplus x y^2 \oplus y^3$$

- The tropical vanishing set (the roots) $V(f)$ of f is

  $V(f) = \{\text{min among the terms of } f \text{ is achieved at least twice}\}$

Thank you for your attention!

I hope that was of some help.