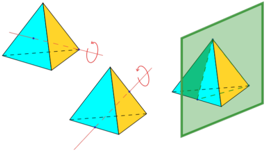
# What is...geometric group theory?

Or: Subfields of mathematics 12

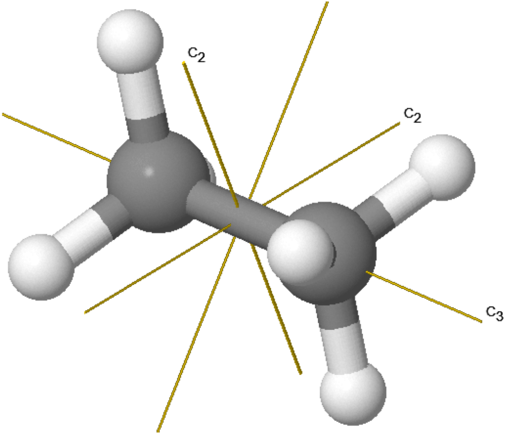
# Groups are symmetries

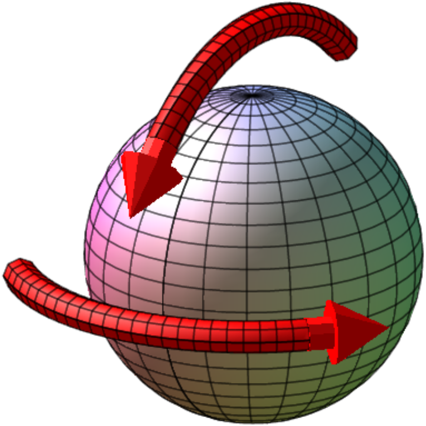
| | Abstract | Incarnation |
|---|---|---|
| Numbers | 3 |  or... |
| Groups | $S_4 = \langle s, t, u \mid \text{some relations} \rangle$ |  or... |

Abstract groups formalize the concept of symmetry

- The study of groups originates in the study of symmetries
- Early example: permutation groups acting on roots of polynomials
- An abstract group is tied to its symmetries

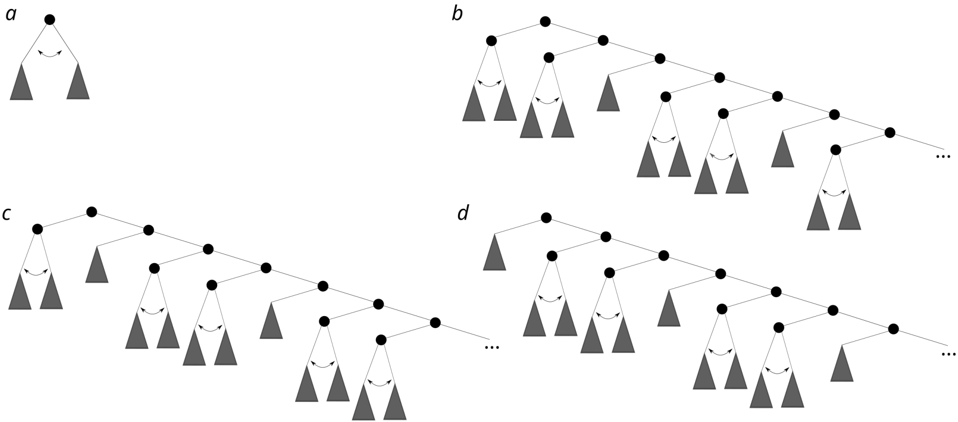
## Finite and continuous



- Finite groups = symmetries of discrete objects
- Lie groups = symmetries of continuous objects
- These are two main classes of groups studied early on

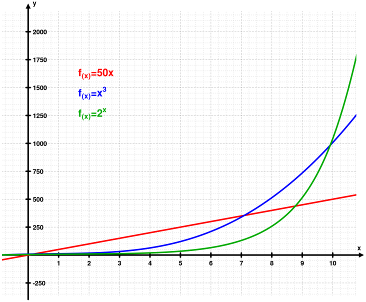
## And all the others?




- Let $a, b, c, d$ act on an infinite binary tree as above
- Grigorchuk group $G$ is the group generated by these symmetries
- $G$ is neither finite nor continuous – how to study such groups?

## Enter, the theorem

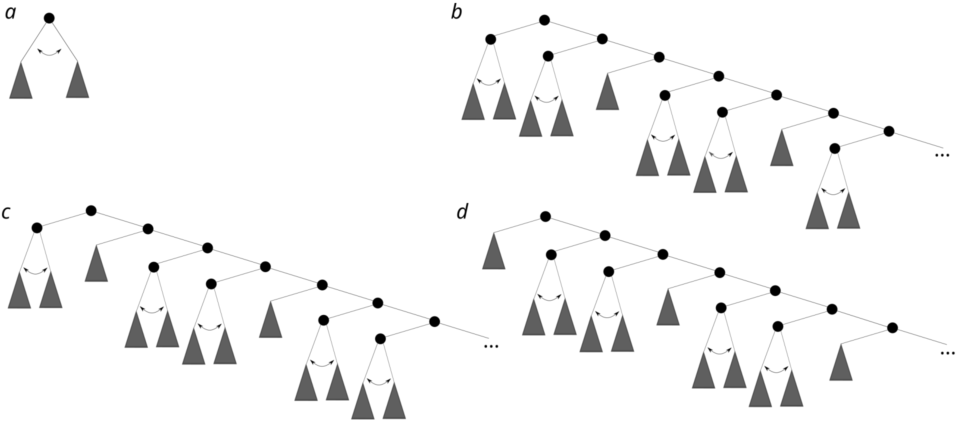
A fg group has polynomial growth $\Leftrightarrow$ it has a nilpotent subgroup of finite index




Here "growth" = the growth of $f(n) = \#$ of elements of word length $n$

- Geometric group theory uses techniques from "geometry" to study groups
- "Geometry" = analytic methods, metric spaces, topologies etc.
- Geometric group theory answers similar questions!

# Intermediate growth




- Example Free groups $F_g$ for $g \geq 2$ are of exponential growth
- Intermediate growth $=$ both superpolynomial and subexponential – do groups with this growth exists?
- Theorem Grigorchuk's group has intermediate growth $e^{n^{0.504}} \leq growth \leq e^{n^{0.7674}}$

**Thank you for your attention!**

I hope that was of some help.