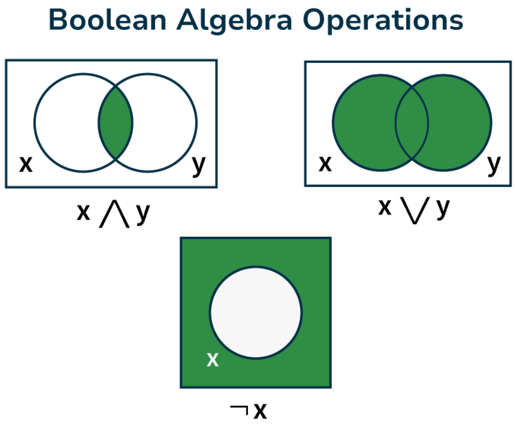
What is...order theory?

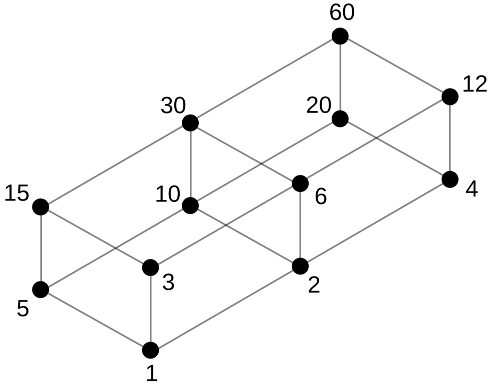
Or: Subfields of mathematics 13

Universal algebra (UA)



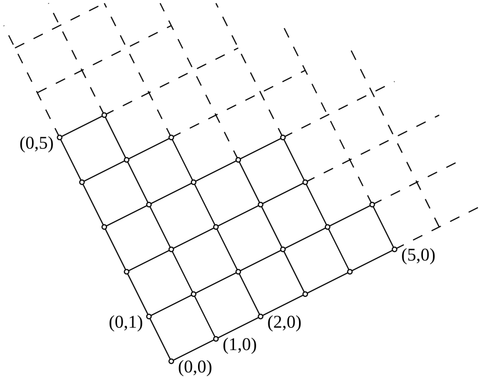
- The study of UA originates in early computer science
- Early example: Boolean algebra = something that mimics true/false logic
- Order theory (OT) $\subset$ UA that focuses on order relations e.g. $x \wedge y < x$

Hasse diagram



- Partial ordered set = a set with $\leq$ satisfying reflexivity, antisymmetry and transitivity
- Example The divisors of 60 with $x \leq y$ if x divides y
- A first tool from OT: a Hasse diagram illustrates partial orders

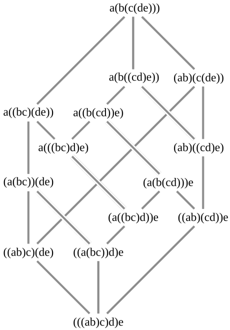
Lattices



- Lattice = partial ordered set with meets $\wedge$ and joins $\vee$
- Example (above) The lattice $\mathbb{N}^2$ ordered componentwise
- Example There is also $\mathbb{N}^2$ with lexicographical order

8 **Enter, the theorem**

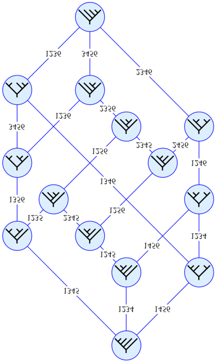
The Tamari lattice is a lattice of size C_n



Vertices = groupings of n elements, $\leq$ application of the rightwards $(ab)c = a(bc)$

- C_n = the nth Catalan number
- The Tamari lattice has several different incarnations
- Order theory answers similar questions!

Associativity and trees



- Tamari lattice = lattice of binary trees up to rotation
- The count of vertices is again given by the Catalan numbers
- There are many more structures that are counted by Catalan numbers

Thank you for your attention!

I hope that was of some help.