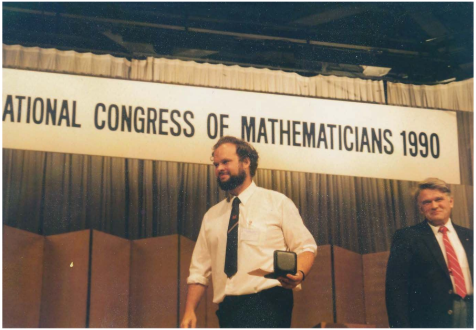
What is...quantum topology?

Or: Subfields of mathematics 16

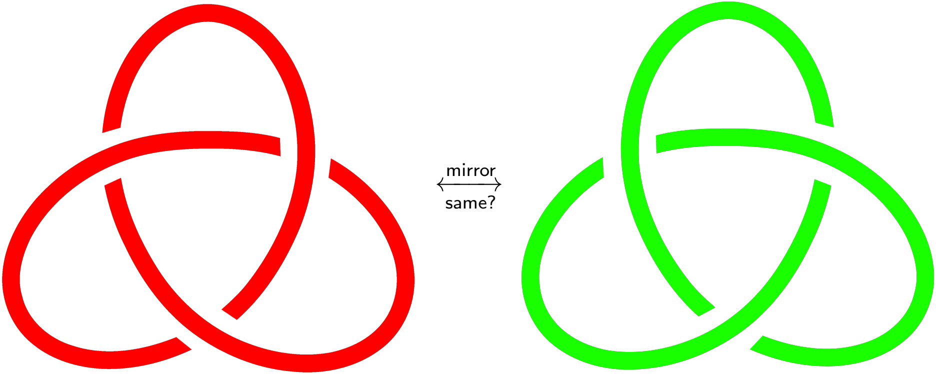
The birth of quantum topology

Jones was awarded the Fields Medal at Kyoto in 1990 for these breakthroughs.



- Kyoto 1990 Jones receives the fields medal
- Quote "Jones discovered an astonishing relationship between von Neumann algebras and geometric topology. As a result, they found a new polynomial invariant for knots and links in 3-space."
- Why this is great: solving old conjectures and and making new connections

Are these the same?



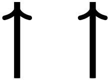
- Problem Deciding whether two knot projections are the same knot is difficult
- Task Find an invariant. Sounds easy? Well, most knot invariants are pretty bad...so: find a 'good' knot invariant
- Before Jones no knot invariant could distinguish the above mirrors

The Jones polynomial

There is a polynomial invariant of oriented knots and link

$V(\_)$: knots and link $\to \mathbb{Z}[q, q^{-1}]$

satisfying the Skein relations

$q^{-1} \cdot V\Big($  $\Big) - q \cdot V\Big($  $\Big) = (q^{1/2} - q^{-1/2}) \cdot V\Big($  $\Big)$

- V is characterized by $V(\text{unknot}) = 1$ and the Skein relations
- $V(\text{alternating})$ is an alternating polynomial
- $V(L) =_{q \leftrightarrow q^{-1}} V(\text{mirror of } L)$
  $V(\text{trefoil}) = -q^4 + q^3 + q$, so the trefoil is not equal to its mirror

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- From a worms perspective, V is powerful and easy at the same time
- From a birds perspective, V created a new field of mathematics, quantum topology, connecting various branches of modern mathematics and the sciences

Enter, the theorem

The Jones polynomial arises via many constructions, e.g.:

(i) Via subfactors

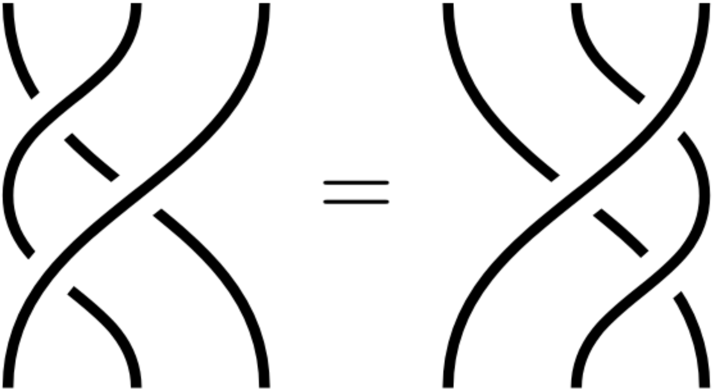
(ii) Via Hecke algebras

(iii) Via quantum groups (QG)

(iv) Many more!

- Many diverse fields! Subfactors come up in the study of von Neumann algebras, Hecke algebras arise from modular forms and representation theory, quantum groups were born trying to solve the Yang–Baxter equation
- Quantum topology makes similar connections!

Yang–Baxter equation (YBE)



- YBE A certain ‘braid-like’ equation from statistical mechanics
- There are (essentially) three types of solutions:
  - Rational $\rightsquigarrow$ Yangian QG
  - Trigonometric $\rightsquigarrow$ affine QG
  - Elliptic $\rightsquigarrow$ elliptic QG

Thank you for your attention!

I hope that was of some help.