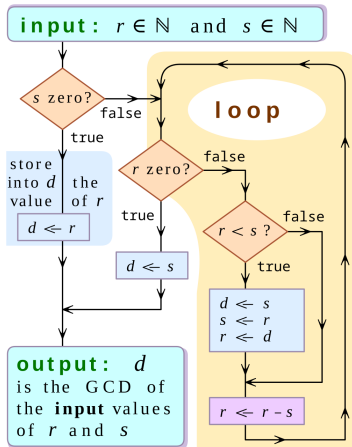


What is...complexity theory?

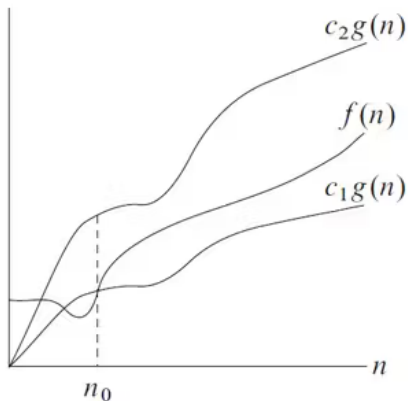
Or: Subfields of mathematics 19

Algorithms rule the world



- ▶ **Algorithm** = a finite sequence of mathematical instructions
- ▶ These are **perfect** for a computer; so they are everywhere
- ▶ **Crucial question** How difficult is it to use algorithm XYZ?

Landau–Bachmann notation




-
- ▶ Above Landau–Bachmann notation: Θ (both), Ω (bottom), O (top)
 - ▶ Task Find, say, $f(n) \in O(\text{nice function})$ for $f(n) =$ function for algorithm
 - ▶ Example $f(n) =$ time complexity, space complexity etc. of the fixed algorithm

Example: multiplication algorithms

Karatsuba



The diagram shows two 4-digit numbers, 2925 and 6872, being decomposed into two 2-digit numbers each. The first number 2925 is split into 'a' (29) and 'b' (25). The second number 6872 is split into 'c' (68) and 'd' (72). Red boxes and lines illustrate this decomposition.



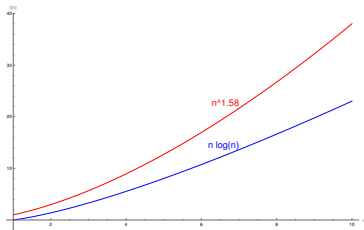
python™

$$\begin{aligned}xy &= (a \times 10^{n/2} + b)(c \times 10^{n/2} + d) \\ &= ac \times 10^n + (ad + bc) \times 10^{n/2} + bd\end{aligned}$$

- ▶ Standard integer multiplication is in $O(n^2)$, with n = number of digits
- ▶ Karatsuba integer multiplication is in $O(n^{\log_2 3} \approx 1.58)$
- ▶ This is great and implemented in many programming languages

Enter, the theorem

Actually, integer multiplication is in $O(n \log n)$



March 2021

Integer multiplication in time $O(n \log n)$

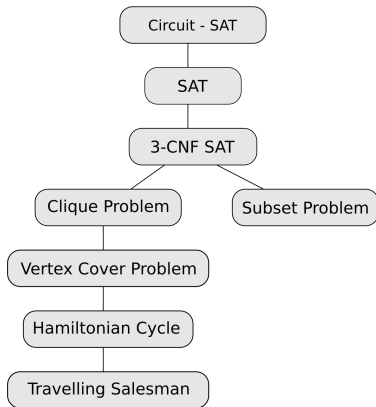
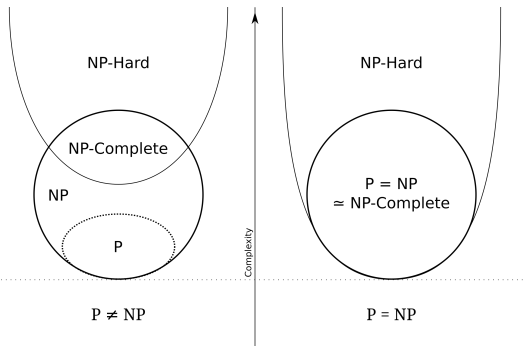
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- ▶ It is even conjectured that this is optimal
- ▶ Catch This is not practical due to overhead
- ▶ Complexity theory answers similar questions!

A more theoretic framework



- ▶ Complexity theory also studies complexity classes
- ▶ Example NP complete problems \approx “all potential algorithm are exponential”
- ▶ Example (right) Several problems that are NP complete \approx “just difficult”

Thank you for your attention!

I hope that was of some help.