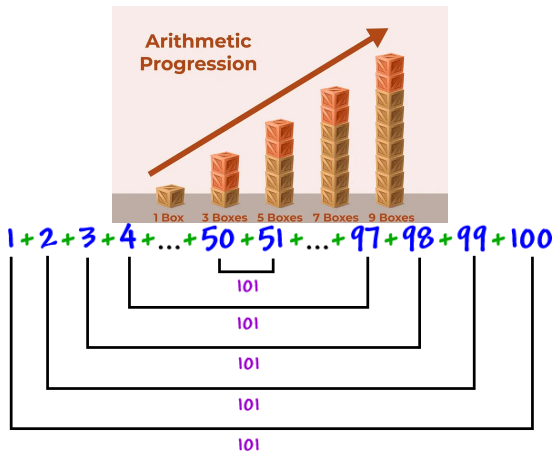


What is...additive combinatorics?

























Or: Subfields of mathematics 2

Arithmetic progression



- ▶ Arithmetic progression (AP) = constant difference between terms
- ▶ Example "Little Gauss"
- ▶ APs are studied since the early days of math

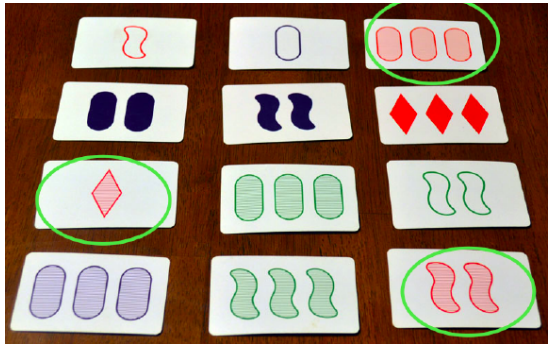
Additive questions

							
Z_1	Z_2	Z_3	Z_4	Z_5	$Z_6 = Z_3 \times Z_2$	Z_7	Z_8
							
Z_9	$Z_{10} = Z_5 \times Z_2$	Z_{11}	$Z_{12} = Z_4 \times Z_3$	Z_{13}	$Z_{14} = Z_7 \times Z_2$	$Z_{15} = Z_5 \times Z_3$	Z_{16}
							
Z_{17}	$Z_{18} = Z_9 \times Z_2$	Z_{19}	$Z_{20} = Z_5 \times Z_4$	$Z_{21} = Z_7 \times Z_3$	$Z_{22} = Z_{11} \times Z_2$	Z_{23}	$Z_{24} = Z_8 \times Z_3$

- ▶ Additive combinatorics studies AP, very broadly interpreted
- ▶ Example For finite subsets $A, B \subset \mathbb{Z}/p\mathbb{Z}$ what can be said about $|A + B|$?
- ▶ Example answer One has $|A + B| \geq \min(|A| + |B| - 1, p)$

The game Set

	3 different shadings			3 different colours			3 different shadings		
3 different numbers of symbols	1 diamond	2 diamonds	3 diamonds	1 diamond	2 diamonds	3 diamonds	1 diamond	2 diamonds	3 diamonds
3 different symbols	1 oval	1 squiggle	1 diamond	1 oval	1 squiggle	1 diamond	1 oval	1 squiggle	1 diamond
3 different numbers of symbols	1 oval	2 ovals	3 ovals	1 oval	2 ovals	3 ovals	1 oval	2 ovals	3 ovals
3 different symbols	1 oval	1 squiggle	1 diamond	1 oval	1 squiggle	1 diamond	1 oval	1 squiggle	1 diamond
3 different numbers of symbols	1 oval	2 ovals	3 ovals	1 oval	2 ovals	3 ovals	1 oval	2 ovals	3 ovals
3 different symbols	1 oval	1 squiggle	1 diamond	1 oval	1 squiggle	1 diamond	1 oval	1 squiggle	1 diamond
3 different numbers of symbols	1 oval	2 ovals	3 ovals	1 oval	2 ovals	3 ovals	1 oval	2 ovals	3 ovals
3 different symbols	1 oval	1 squiggle	1 diamond	1 oval	1 squiggle	1 diamond	1 oval	1 squiggle	1 diamond
3 different numbers of symbols	1 oval	2 ovals	3 ovals	1 oval	2 ovals	3 ovals	1 oval	2 ovals	3 ovals
3 different symbols	1 oval	1 squiggle	1 diamond	1 oval	1 squiggle	1 diamond	1 oval	1 squiggle	1 diamond

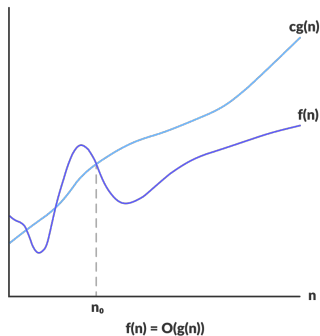


- ▶ **Set** = card game with $3^4 = 81$ cards that vary in four features
- ▶ **Goal** Find 'sets' where all features are the same or different
- ▶ **Enter, APs** Cards correspond to points of $(\mathbb{Z}/3\mathbb{Z})^4$, sets are 3-APs

Enter, the theorem

Every 3-AP-free subset of $(\mathbb{Z}/3\mathbb{Z})^n$ has size $O(3^n/n)$

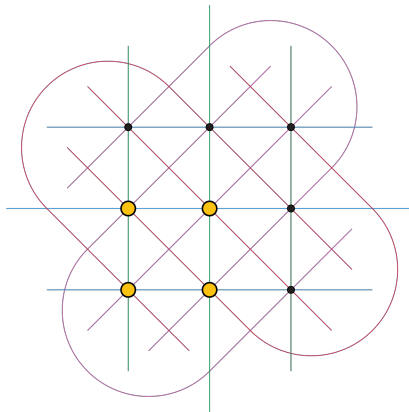
Big O notation:



This is Roth's theorem in $(\mathbb{Z}/3\mathbb{Z})^n$

- ▶ This is a combinatorial version of Roth's theorem (a classic in number theory)
- ▶ Additive combinatorics answers similar questions!

This is also finite geometry



-
- ▶ **Cap set** = subset of $(\mathbb{Z}/3\mathbb{Z})^n$ where no three elements sum to the zero vector
 - ▶ **Example (above)** 9 points, 12 lines in $(\mathbb{Z}/3\mathbb{Z})^2$, and a (yellow) cap set
 - ▶ **In Set** a cap set is of size ≤ 20

Thank you for your attention!

I hope that was of some help.