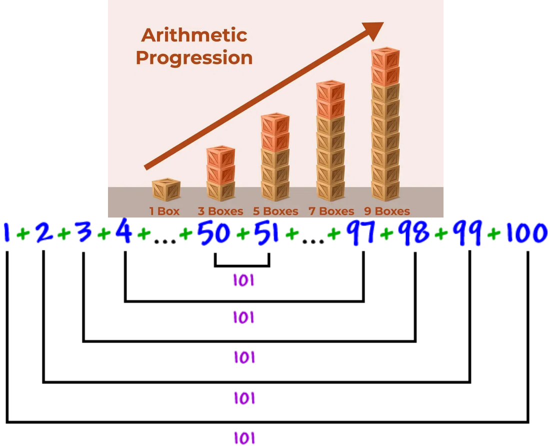
What is...additive combinatorics?

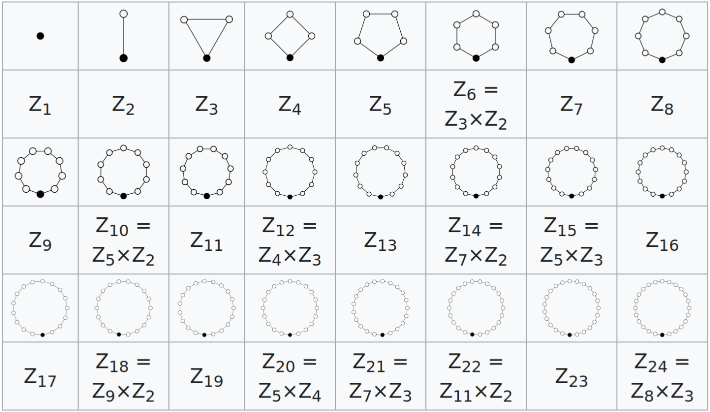
Or: Subfields of mathematics 2

Arithmetic progression



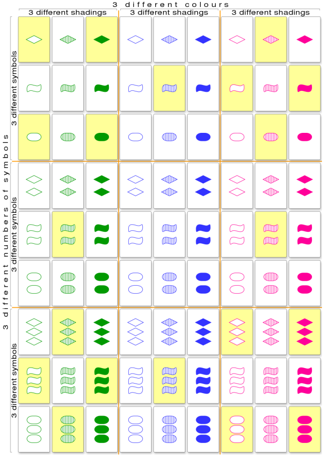
- Arithmetic progression (AP) = constant difference between terms
- Example "Little Gauss"
- APs are studied since the early days of math

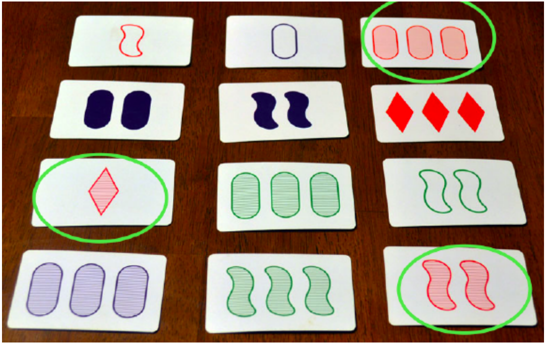
Additive questions



- **Additive combinatorics** studies AP, very broadly interpreted
- **Example** For finite subsets $A, B \subset \mathbb{Z}/p\mathbb{Z}$ what can be said about $|A+B|$?
- **Example answer** One has $|A+B| \geq \min(|A|+|B|-1, p)$

The game Set



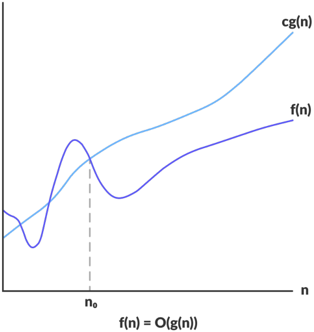


- Set = card game with $3^4 = 81$ cards that vary in four features
- Goal Find 'sets' where all features are the same or different
- Enter, APs Cards correspond to points of $(\mathbb{Z}/3\mathbb{Z})^4$, sets are 3-APs

Enter, the theorem

Every 3-AP-free subset of $(\mathbb{Z}/3\mathbb{Z})^n$ has size $O(3^n/n)$

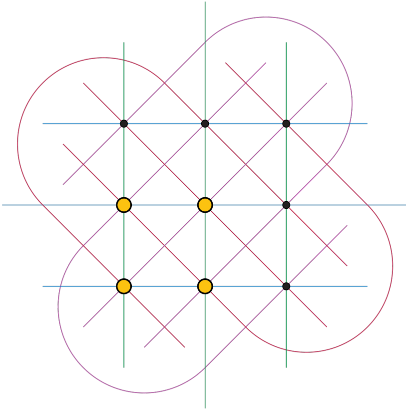
Big O notation:



This is Roth's theorem in $(\mathbb{Z}/3\mathbb{Z})^n$

- This is a combinatorial version of Roth's theorem (a classic in number theory)
- Additive combinatorics answers similar questions!

This is also finite geometry



- Cap set = subset of $(\mathbb{Z}/3\mathbb{Z})^n$ where no three elements sum to the zero vector
- Example (above) 9 points, 12 lines in $(\mathbb{Z}/3\mathbb{Z})^2$, and a (yellow) cap set
- In Set a cap set is of size ≤ 20

Thank you for your attention!

I hope that was of some help.