What is...game theory?

Or: Subfields of mathematics 20

Game: hawk vs. dove

- ▶ Hawk strategy Always attack

- ▶ Dove strategy Retreat unless you see a dove

- ▶ Game Reward = V, damage = C
    - ▶ Hawk-meets-hawk: both fight, get some of the reward and also hurt
    - ▶ Hawk-meets-dove: hawk gets the reward
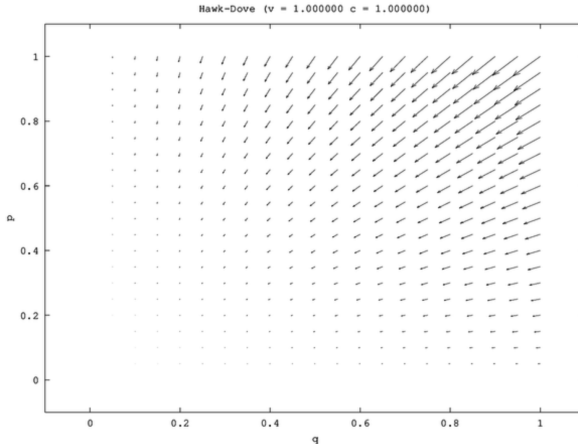   - ▶ Dove-meets-dove: both share the reward

Payoff matrix

	Hawk	Dove
Hawk	(V−C)/2, (V−C)/2	V, 0
Dove	0, V	V/2, V/2

▶ Payoff matrix = a way to express the result of players' choices

▶ Explicit example For $V = 2$ and $C = 4$ we get

$$\begin{pmatrix} -1, -1 & 2, 0 \\ 0, 2 & 1, 1 \end{pmatrix}$$

▶ Question How does the system change depending on V, C?

Evolution of the system

Hawk-Dove (v = 1.000000 c = 1.000000)

- Above The best response mapping for players p and q

- The percentage ⤳ player chooses dove

- What we are interested in are fixed points of the system

Enter, the theorem

There are two crucial differences :

(i) In the one population model (a single population which plays against itself) every initial population proportion (except all hawk and all dove) converge to the mixed strategy below

(ii) In the two population model (two populations play against each other) the only stable state (pure strategy) is below
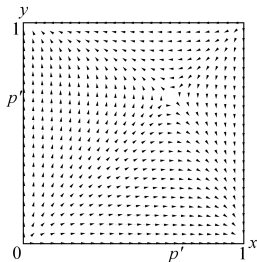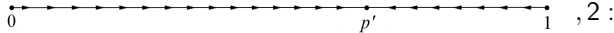
▶ How does this applies to real-world politics? Unclear ☺

▶ Game theory answers similar questions!

Prisoner's dilemma

- ▶ Above For $C \leq V$ this becomes the famous prisoner's dilemma

- ▶ Payoff matrix could be

$$\begin{pmatrix} -1, -1 & -3, 0 \\ 0, -3 & -2, -2 \end{pmatrix}$$

- ▶ Tiny catch Here the 'reward' needs to be negative

Thank you for your attention!

I hope that was of some help.