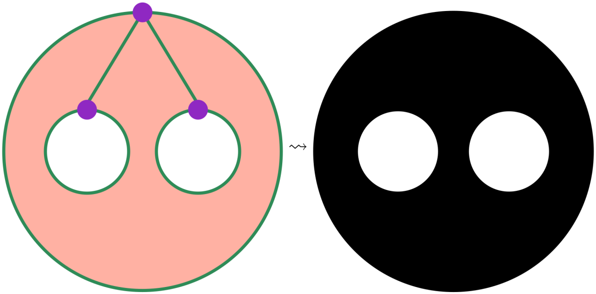
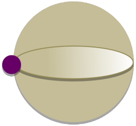
What is...discrete geometry?

Or: Subfields of mathematics 22

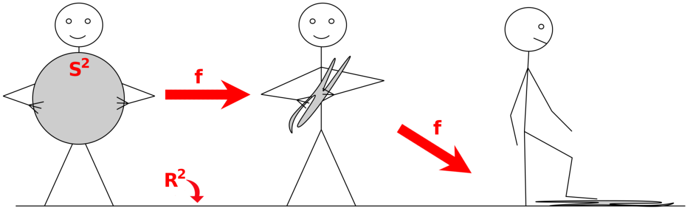
Combinatorics in topology

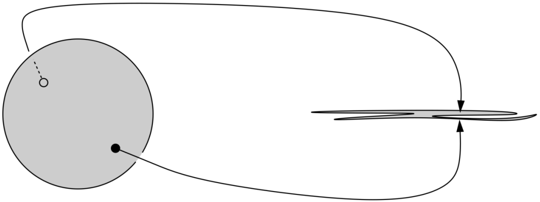




- Combinatorics is part of topology from the very beginning
- Example Cell complexes are combinatorics ways to describe topological spaces
- What about the other way around?

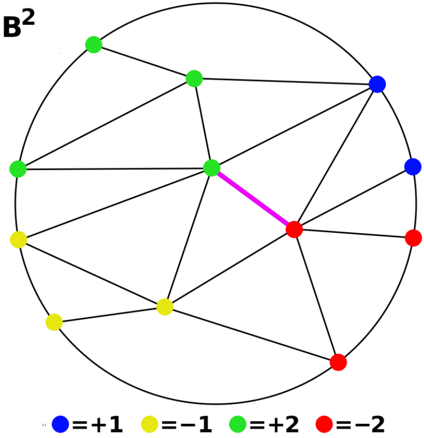
A classic in topology

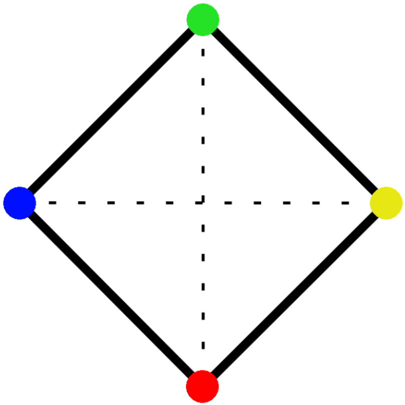




- Theorem For every continues map $f\colon S^n \to \mathbb{R}^n$ $\exists x\colon f(x) = f(-x)$
- This is the celebrated Borsuk–Ulam theorem
- Roughly $\exists$ a pair of opposite locations with the same temperature and pressure

Topology in combinatorics



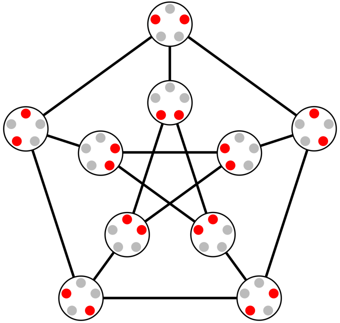


- Theorem Antipodally triangulate B^n, label vertices $\{\pm1, ..., \pm n\}$ in antipodal-symmetric fashion, then $\exists$ edge with vertices labeled $(x, -x)$
- This is the celebrated combinatorial Borsuk–Ulam theorem

Enter, the theorem

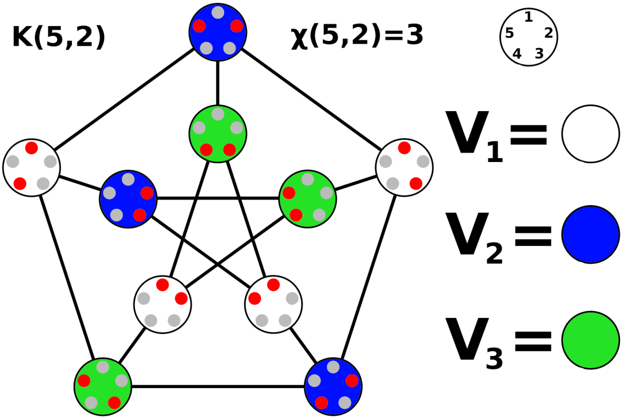
Kneser's conjecture holds (more on the next slide)

$K(5, 2)$:



- Above A Kneser graph $K(n, k)$
- The point The conjecture is purely combinatorial but was first proven with the Borsuk–Ulam theorem
- Discrete geometry answers similar questions with similar methods!

More on Kneser's conjecture



- $K(n,k)$ = vertices k-element subsets of $\{1,...,n\}$, edges between $A \cap B = \emptyset$
- Classical question What is the smallest $\chi(n,k)$ (coloring) such that $K(n,k)$ can be partitioned into $V_1 \dot\cup ... \dot\cup V_\chi$ of intersecting family of k-sets V_i?
- Kneser's conjecture (Aufgabe 360). $\chi(2k+d,k) = d+2$

Thank you for your attention!

I hope that was of some help.