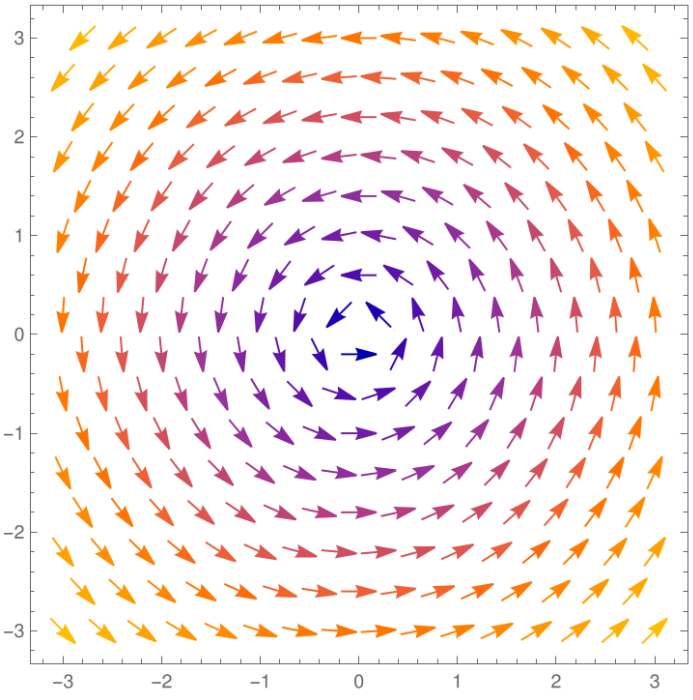
What is...numerical linear algebra?

Or: Subfields of mathematics 27

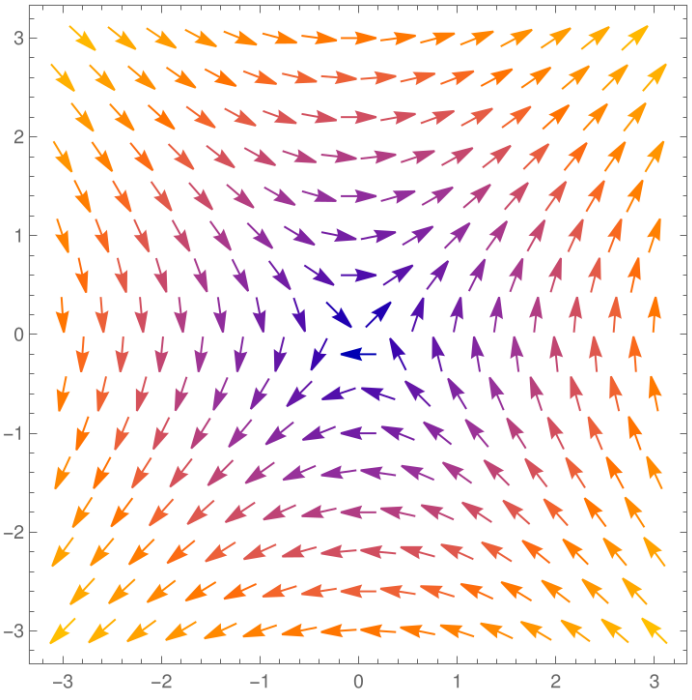
Fixed points and their magnitudes (eigenvalues = EV)

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$



no fixed vector

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$



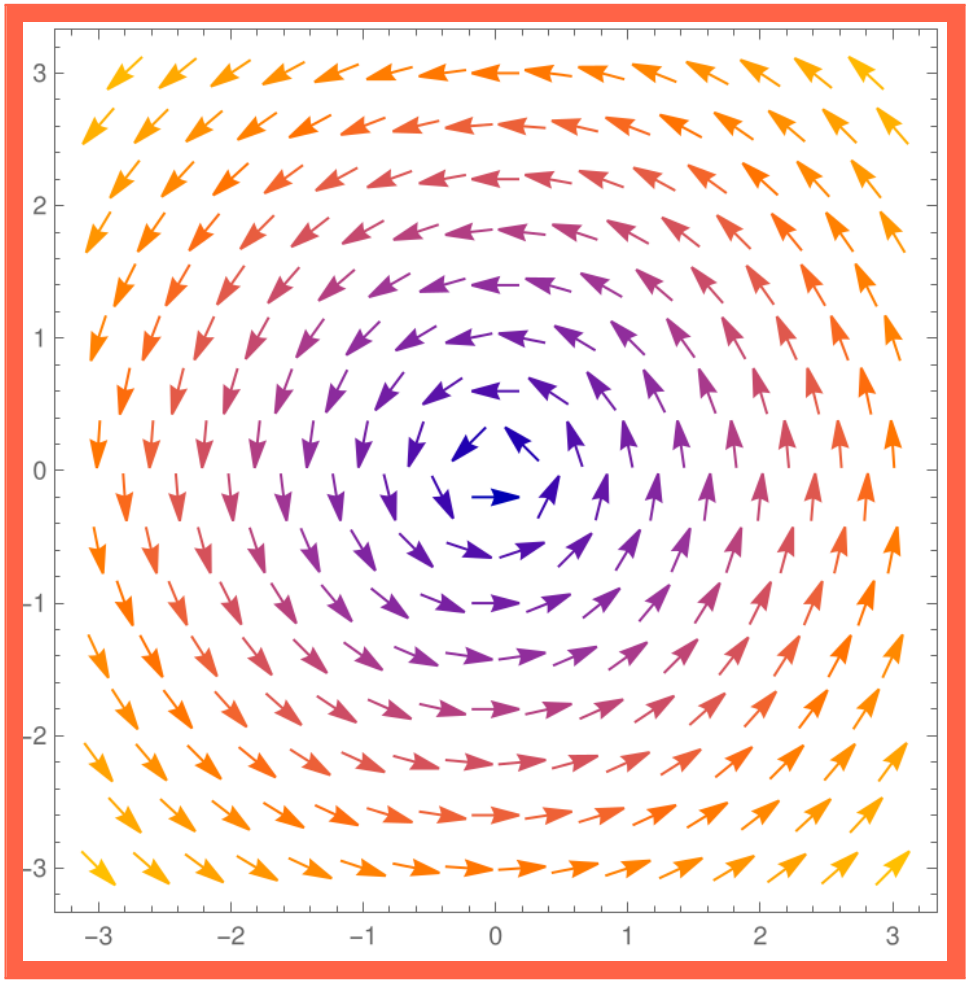
fixed vectors

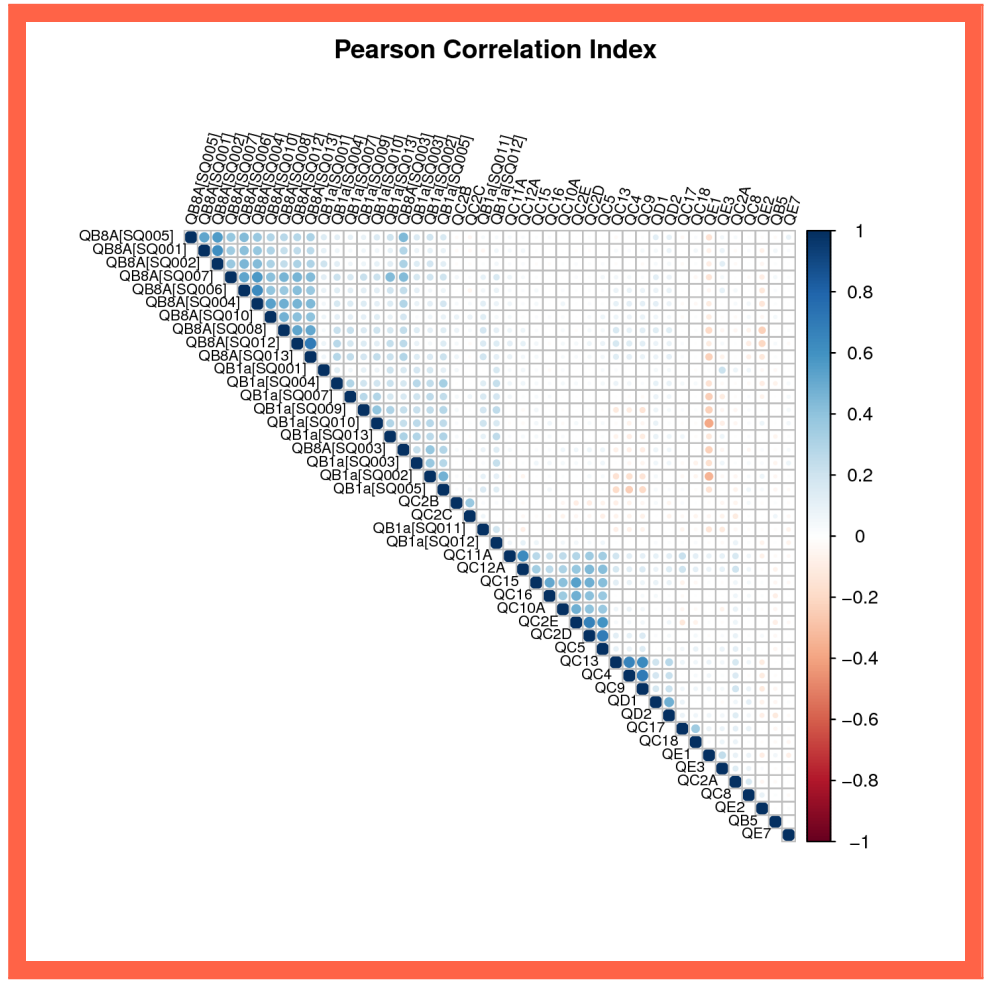
- Above Streamlines of the vector field $M.(x, y)$ for the M as above
- The eigenvector are visible: they correspond to fixed lines
- Eigenvector/values "=" fixed lines/magnitudes of them

Real-world applications? Sure!

- The 1940 Tacoma Narrows Bridge dramatically collapsed probably due to resonance effects
- Resonance problems "are" EV problems!
- Crucial Is it easy to calculate EVs in a "good" way?

QR = no-changes-times-easy





- Problem "Roots of $\det(M - x \cdot Id)$" is not a great way to compute EVs
- Better Use the $M = QR$ decomposition, $Q =$ orthogonal, $R =$ upper $\triangle$
- orthogonal "=" rotation $\rightsquigarrow$ no EV change (not quite true); upper $\triangle$ $\rightsquigarrow$ easy EVs

Enter, the theorem

QR iteration (upon certain conditions):

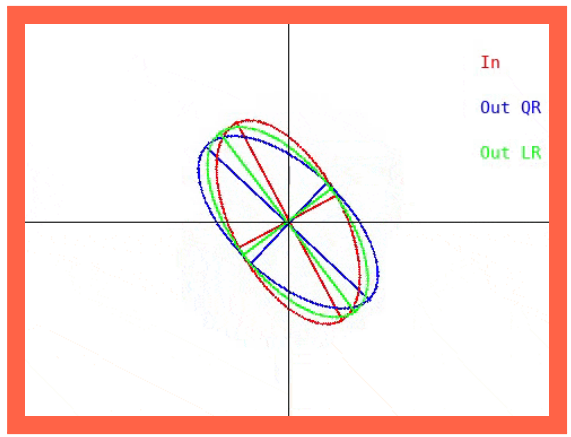
(i) Write $M_0 = M$ Start

(ii) Compute $M_k = Q_k R_k$ and set $M_{k+1} = R_k Q_k$ Step

(iii) M_k converges to a upper triangular matrix M_∞ for $k \to \infty$ Finish

M_∞ has same EVs as M

- Upon correct implementation, this has way more upshots, *e.g.* is quite fast, is numerically stable, *etc.*



- Numerical linear algebra answers similar questions!

Algorithms of the century

- Metropolis Algorithm for Monte Carlo
- Simplex Method for Linear Programming
- Krylov Subspace Iteration Methods
- The Decompositional Approach to Matrix Computations
- The Fortran Optimizing Compiler
- QR Algorithm for Computing Eigenvalues
- Quicksort Algorithm for Sorting
- Fast Fourier Transform
- Integer Relation Detection
- Fast Multipole Method

- Above From the IEEE Computer Society Journal
- No such list can be perfect but that QR iteration made it on it should tell us something ☺

Thank you for your attention!

I hope that was of some help.