

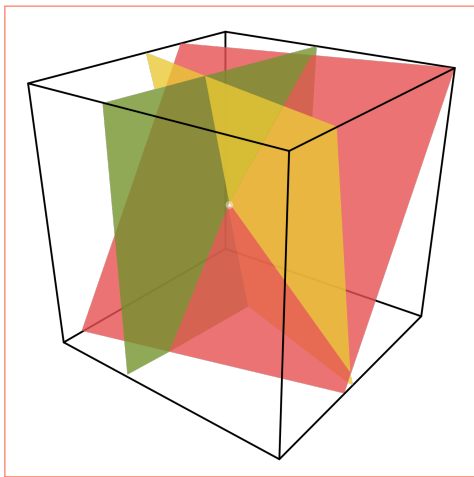
**What is...linear programming?**

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Or: Subfields of mathematics 28

## Systems of linear equations

$$\begin{cases} 3x + 2y - z = 1 \\ 2x - 2y + 4z = -2 \\ -x + \frac{1}{2}y - z = 0 \end{cases} \iff$$

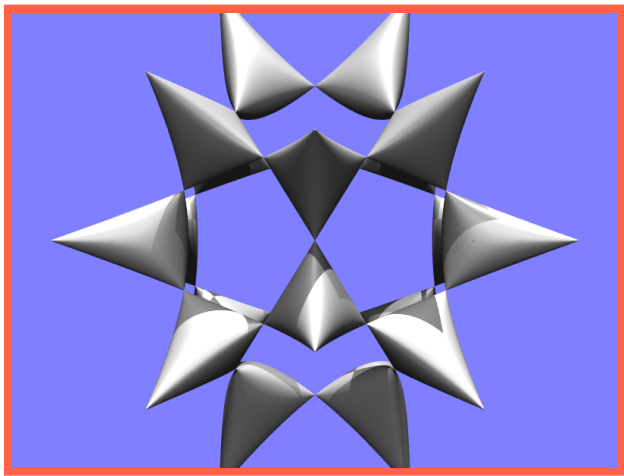


- ▶ Systems of linear equations are remarkably easy to solve
- ▶ These are so predominant that things are modeled to fit into a linear scheme – taking errors into account; Problem: almost nothing is linear

## Systems of nonlinear equations

$$\begin{aligned} &4(\Phi^2 x^2 - y^2) \\ &(\Phi^2 y^2 - z^2) \\ &(\Phi^2 z^2 - x^2) \\ &-(1 + 2\Phi) \\ &(x^2 + y^2 + z^2 - w^2)2w^2 \\ &= 0 \end{aligned}$$

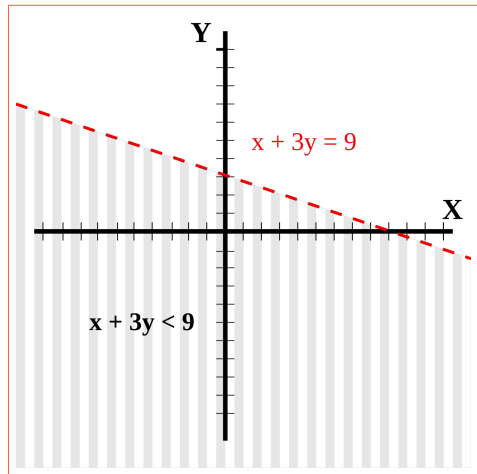
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- ▶ Systems of **nonlinear** equations are **often very difficult** to solve
- ▶ These are so **nasty** that whole subfields of math are dedicated to one type;  
**Problem: they fit to the real-world but are too difficult**

# Systems of linear inequalities

Standard LP problem: Find a vector  $\mathbf{x}$   
that maximizes  $\mathbf{c}^T \mathbf{x}$   
subject to  $A\mathbf{x} \leq \mathbf{b}$   
and  $\mathbf{x} \geq \mathbf{0}$ .



- ▶ Systems of linear inequalities are surprisingly easy to solve
- ▶ This observation started linear programming (LP) which asks to optimize a value subject to linear inequality constraints

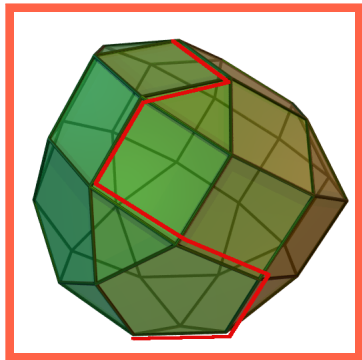
## Enter, the theorem

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There is an algorithm that has polynomial-time average-case complexity and that solves all standard LP problems

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- ▶ The algorithm uses a simplex and is therefore called simplex algorithm



- ▶ The worst case efficiency is however quite bad (exponential)
- ▶ Linear programming (LP) answers similar questions!

# Algorithms of the century



- Metropolis Algorithm for Monte Carlo
- Simplex Method for Linear Programming
- Krylov Subspace Iteration Methods
- The Decompositional Approach to Matrix Computations
- The Fortran Optimizing Compiler
- QR Algorithm for Computing Eigenvalues
- Quicksort Algorithm for Sorting
- Fast Fourier Transform
- Integer Relation Detection
- Fast Multipole Method

▶ Above From the IEEE Computer Society Journal

▶ No such list can be perfect but that QR iteration made it on it should tell us something 😊

**Thank you for your attention!**

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I hope that was of some help.