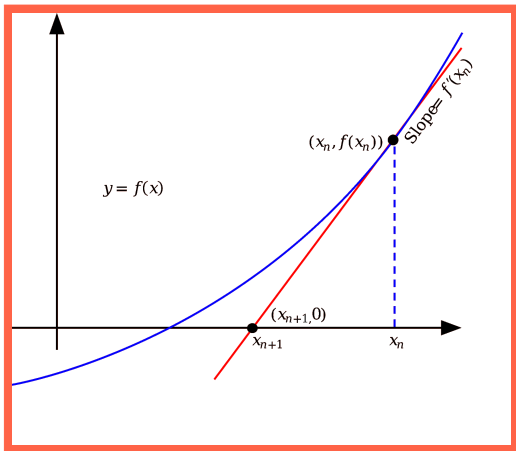**What is...iterative mathematics?**
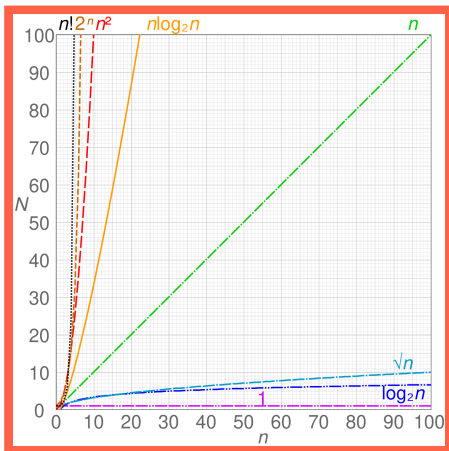
Or: Subfields of mathematics 33

Newton's method


- ▶ Newton's method is a root finding algorithm

- ▶ It uses an iteration – and is a prototypical example of iteration

- ▶ This usually converges fast : the number of accurate digits roughly doubles at each step

Why would we do this?

- Say we want to solve n linear equations – exact algorithms are in $O(n^{2.4})$

- Problems This is actually quite bad, and we need to wait until the calculation is finished

- Iteration advantage We can stop the calculation at any time
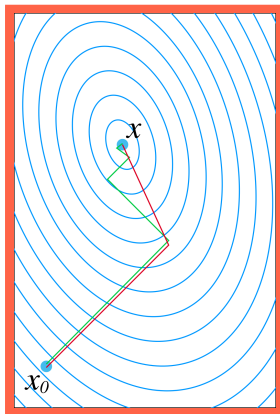
Stability is it!
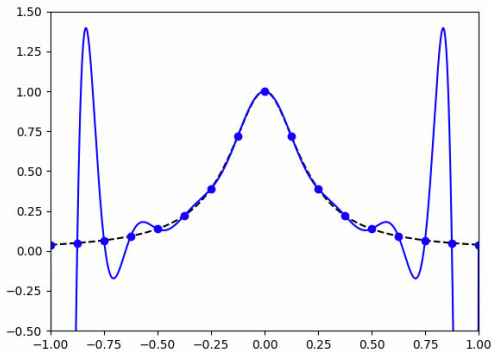
- ▶ The measurement for "How good is an algorithm?" changes

- ▶ For iterations we like to ask

> "How fast and stable does it converge?"

Enter, the theorem

Krylov (subspace) methods solve, e.g., linear equations using iteration

- Krylov subspace $= \mathrm{span}(b, Ab, ..., A^{r-1}b)$ for a matrix A and a vector b
- There are many such methods with good convergence, e.g. gradient methods

- Iterative mathematics answers similar questions!

Algorithms of the century

- Metropolis Algorithm for Monte Carlo
- Simplex Method for Linear Programming
- Krylov Subspace Iteration Methods
- The Decompositional Approach to Matrix Computations
- The Fortran Optimizing Compiler
- QR Algorithm for Computing Eigenvalues
- Quicksort Algorithm for Sorting
- Fast Fourier Transform
- Integer Relation Detection
- Fast Multipole Method

▶ **Above** From the IEEE Computer Society Journal

▶ **No such list can be perfect** but that Krylov methods made it on it should tell us something ☺

Thank you for your attention!

I hope that was of some help.