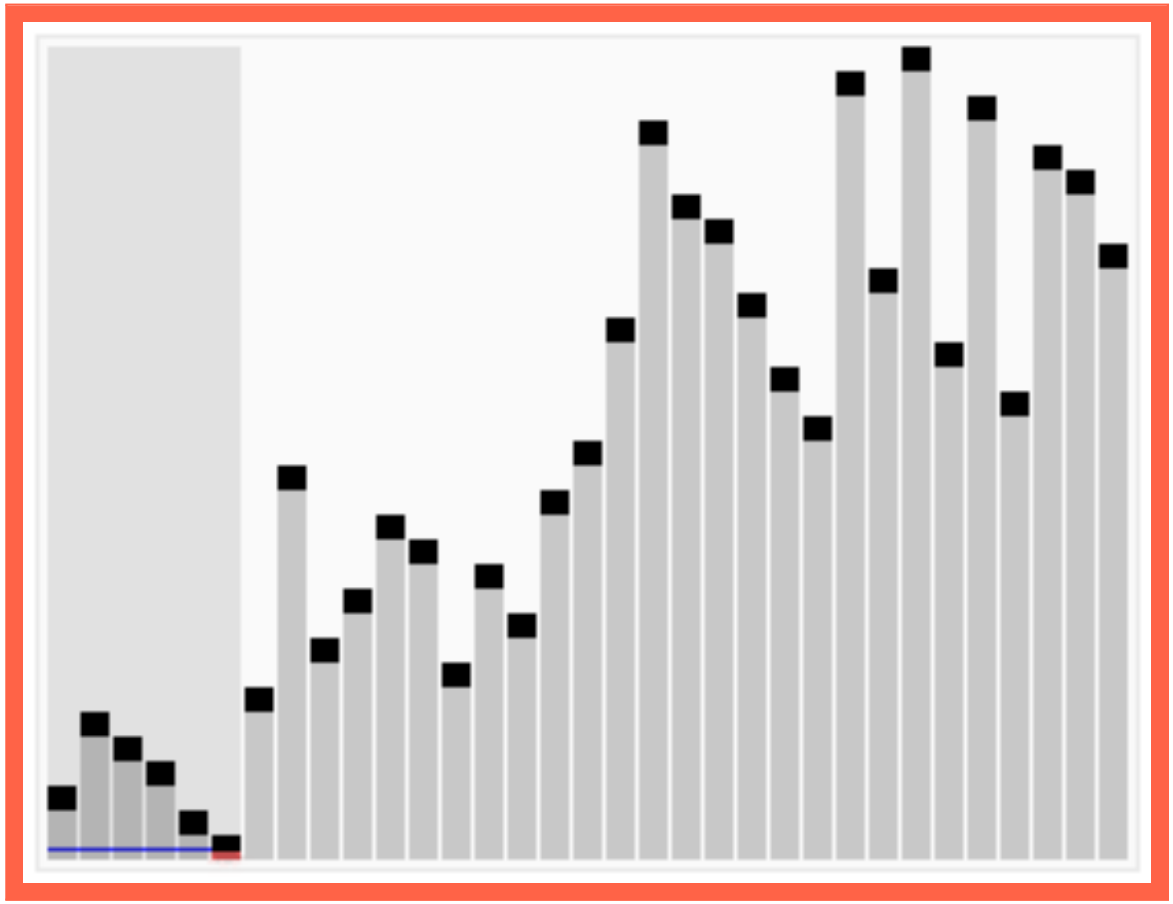
What is...the theory of algorithms?

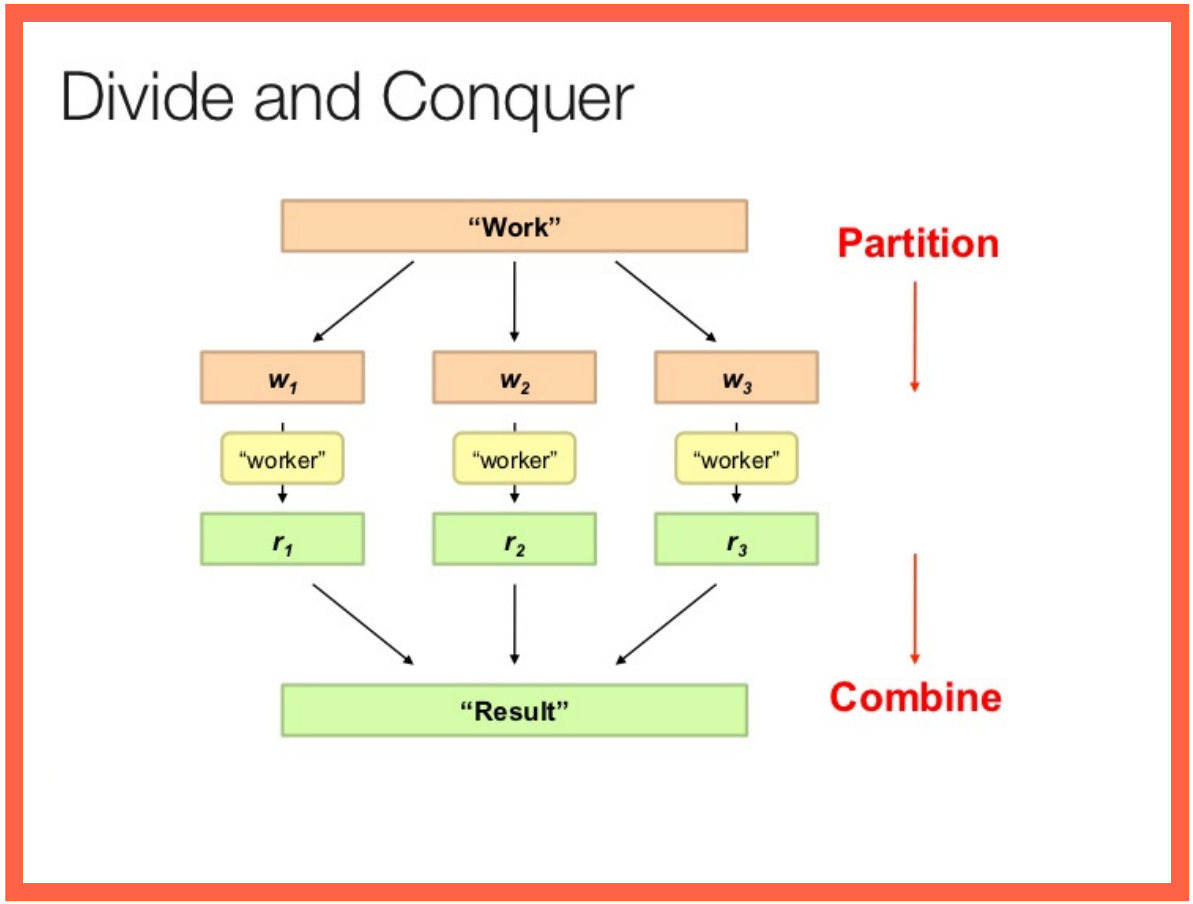
Or: Subfields of mathematics 34

Quicksort



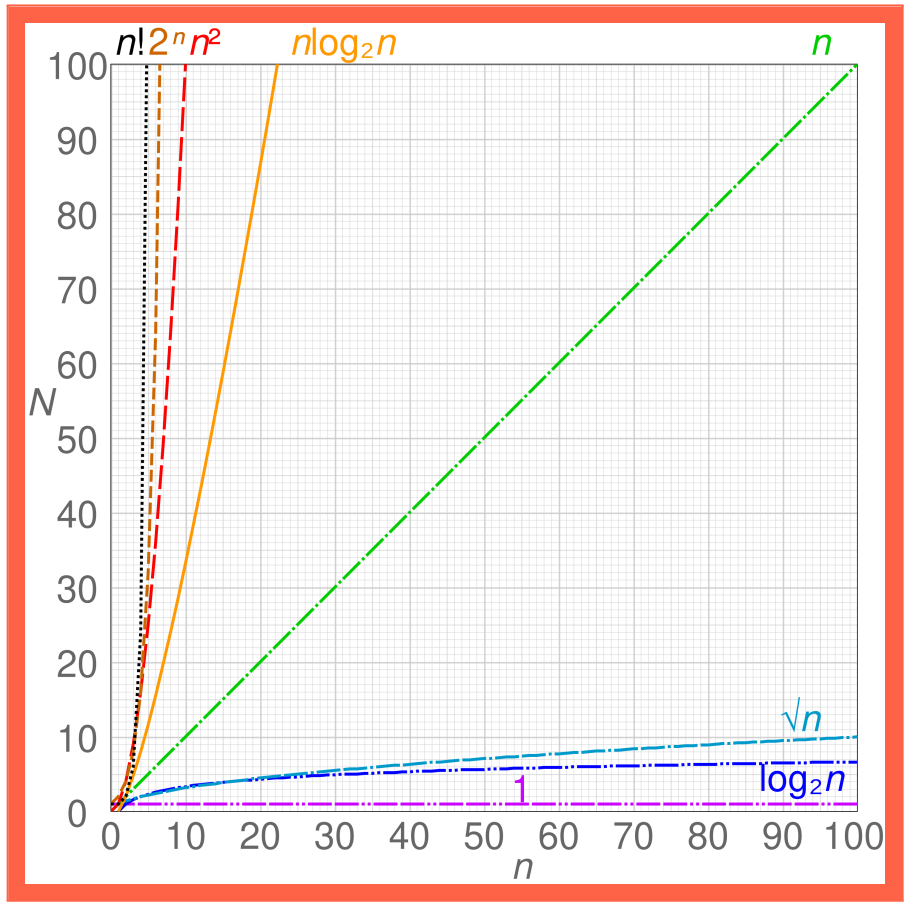
- Quicksort = a family of algorithms for sorting an array, based on a partitioning routine (details depend on the routine)
- Roughly Partition the array into $\leq p$ and $> p$ and apply quicksort recursively
- Here p is e.g. a randomly chosen pivot

Divide and conquer (DAC)



- DAC algorithms = algorithms where a problem is divided into many smaller subproblems
- Power of recursion The complexity often reduces like the dimension of a fractal
- Examples (beyond quicksort) FFT, Strassen, Karatsuba, ...

Analysis of algorithms



- Worst case Vanilla analysis, often doable, asking for the worst-case-scenario
- Best case Left common, often doable, asking for the best-case-scenario
- Average case Rare since difficult in general, asking for the average runtime

Enter, the theorem

Analysis of Quicksort (n=length of array) and other sorting algorithms:

Name	Best	Average	Worst	Memory	Stable
In-place merge sort	—	—	$n \log^2 n$	1	Yes
Block sort	n	$n \log n$	$n \log n$	1	Yes
Smoothsort	n	$n \log n$	$n \log n$	1	No
Heapsort	$n \log n$	$n \log n$	$n \log n$	1	No
Introsort	$n \log n$	$n \log n$	$n \log n$	$\log n$	No
Quicksort	$n \log n$	$n \log n$	n^2	$\log n$	No

- $\Rightarrow$ Sorting is in $O(n \log n)$
- The theory of algorithms answers similar questions!

Algorithms of the century

- Metropolis Algorithm for Monte Carlo
- Simplex Method for Linear Programming
- Krylov Subspace Iteration Methods
- The Decompositional Approach to Matrix Computations
- The Fortran Optimizing Compiler
- QR Algorithm for Computing Eigenvalues
- Quicksort Algorithm for Sorting
- Fast Fourier Transform
- Integer Relation Detection
- Fast Multipole Method

- Above From the IEEE Computer Society Journal
- No such list can be perfect but that quicksort made it on it should tell us something ☺

Thank you for your attention!

I hope that was of some help.