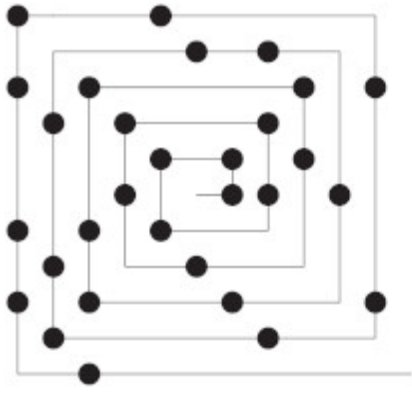
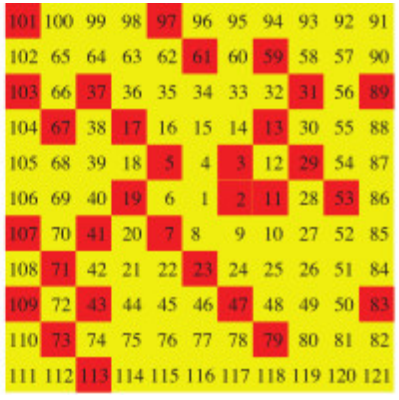
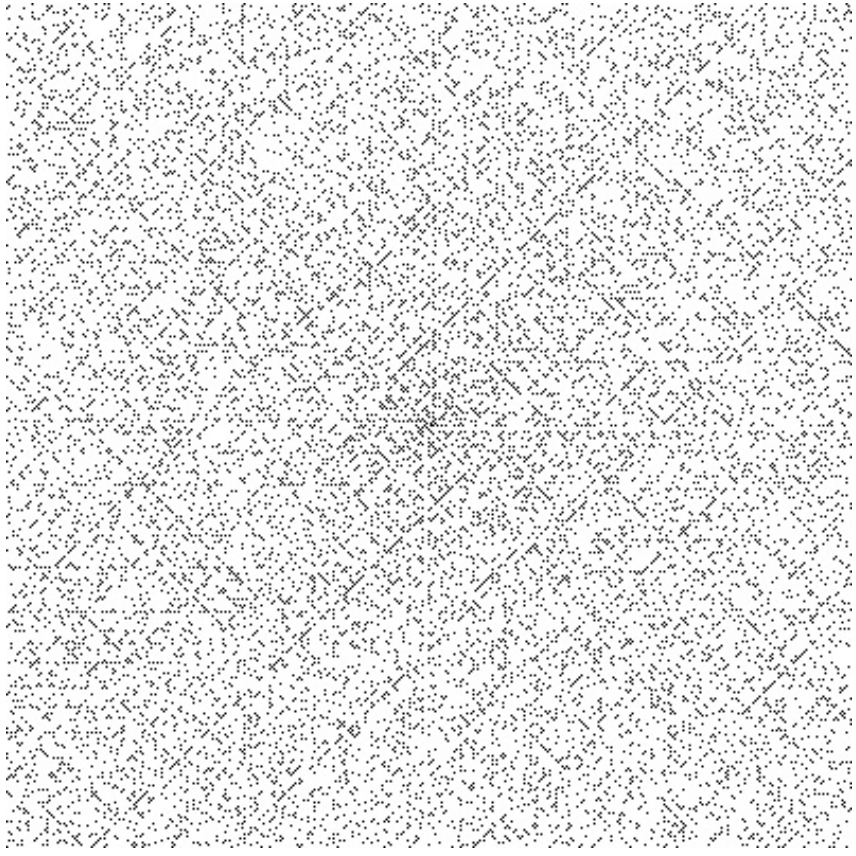
What is...random graph theory?

Or: Subfields of mathematics 5

Primes are kind of random...

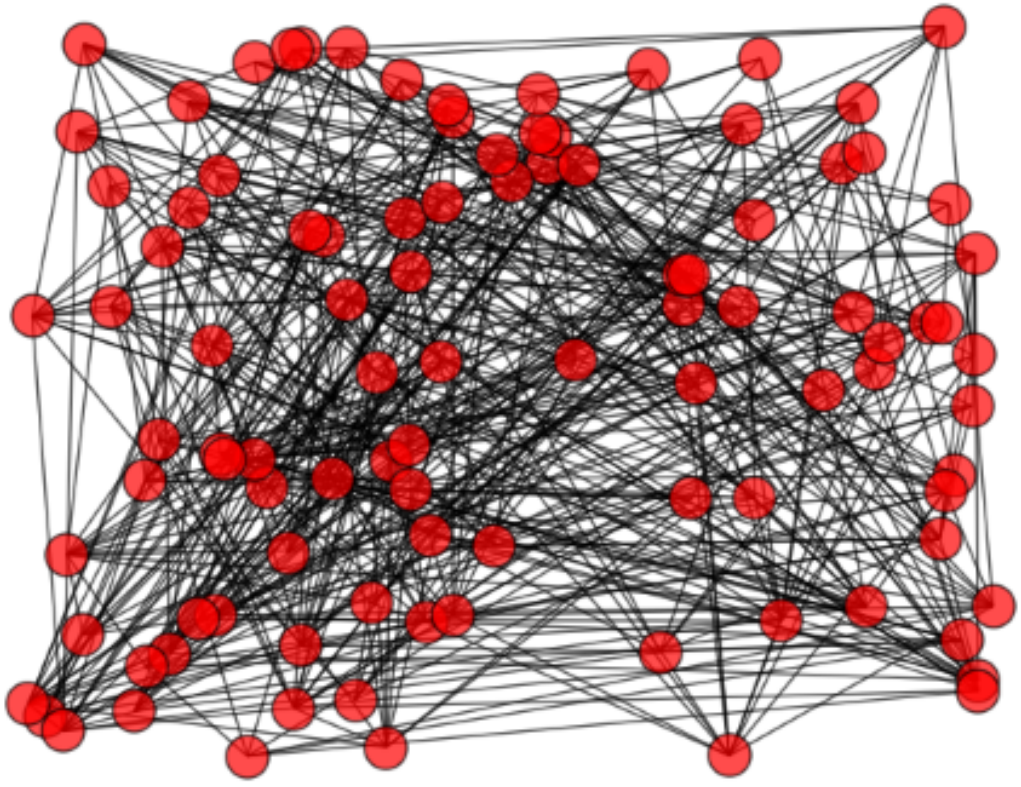






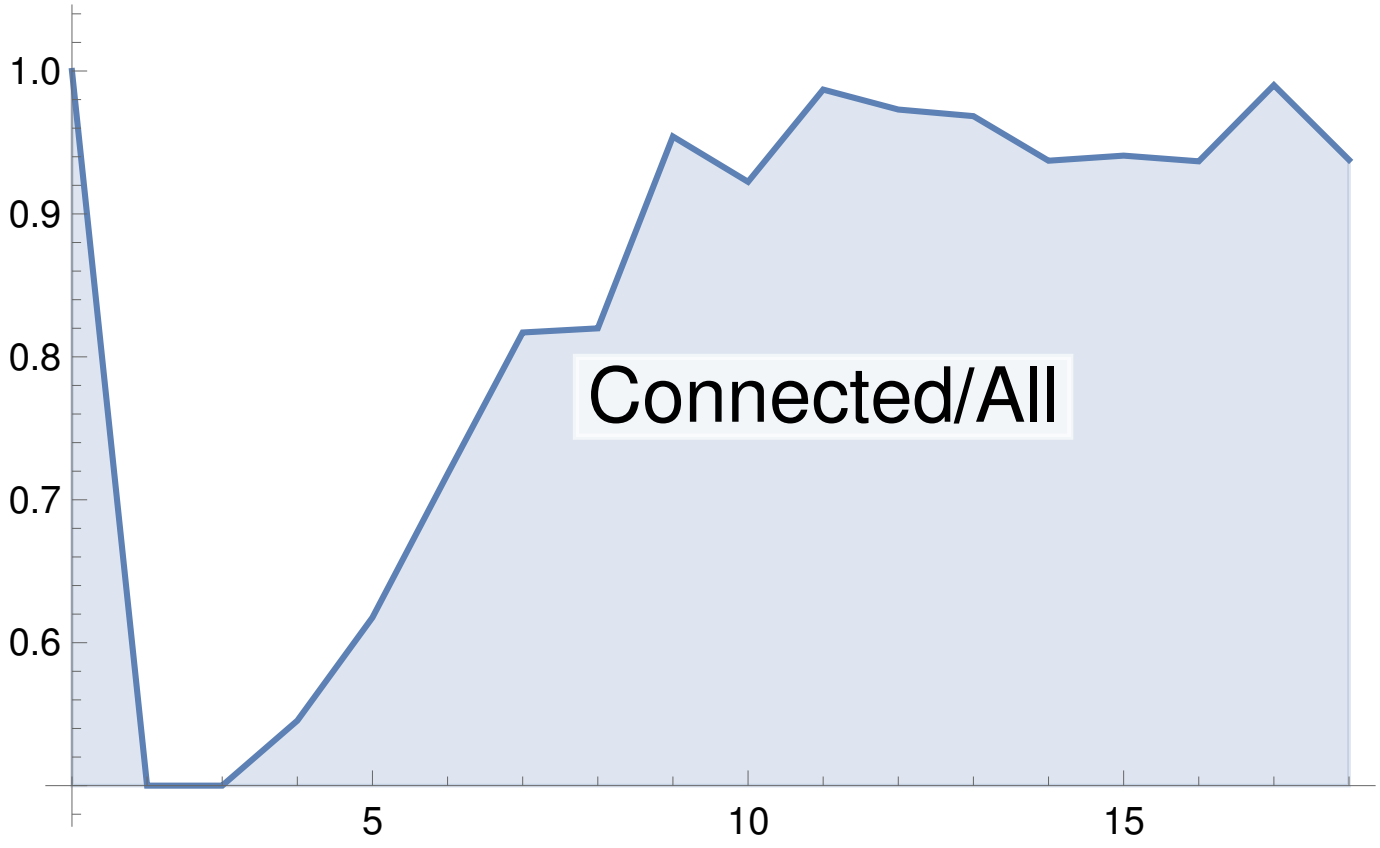
- Prime numbers appear essentially randomly
- Zooming out, they mostly look like noise
- However, also many patterns can be observed

Graphs are kind of random...



- Random graphs = choose edges randomly = "average graphs"
- Zooming out, they mostly look like noise
- However, also many patterns can be observed

Patterns in randomness



- Fact Most graphs have many edges
- Random graphs are almost always connected (=almost all graphs are connected)
- Above # connected graphs / # all graphs

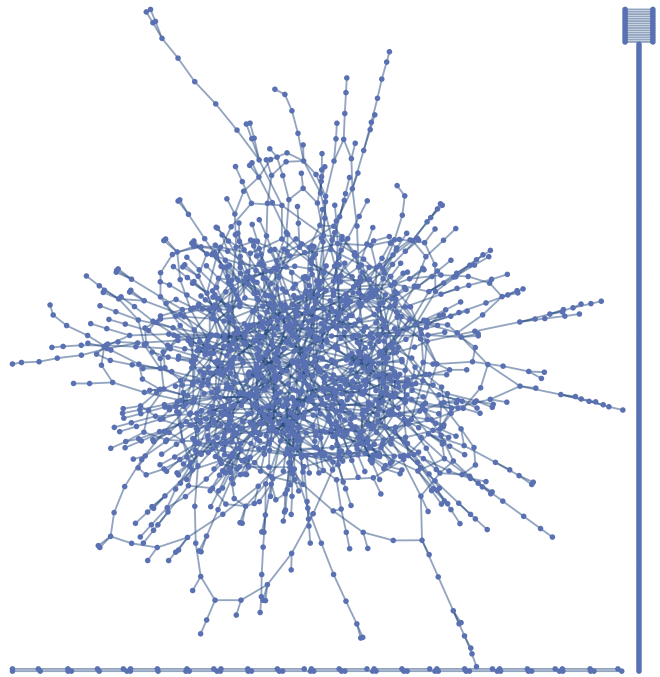
Enter, the theorem

$G_{n,p}$ = random graph with n vertices and probability p to connect two vertices

(i) If p is constant, then $G_{n,p}$ is connected with probability 1 for $n \gg 0$

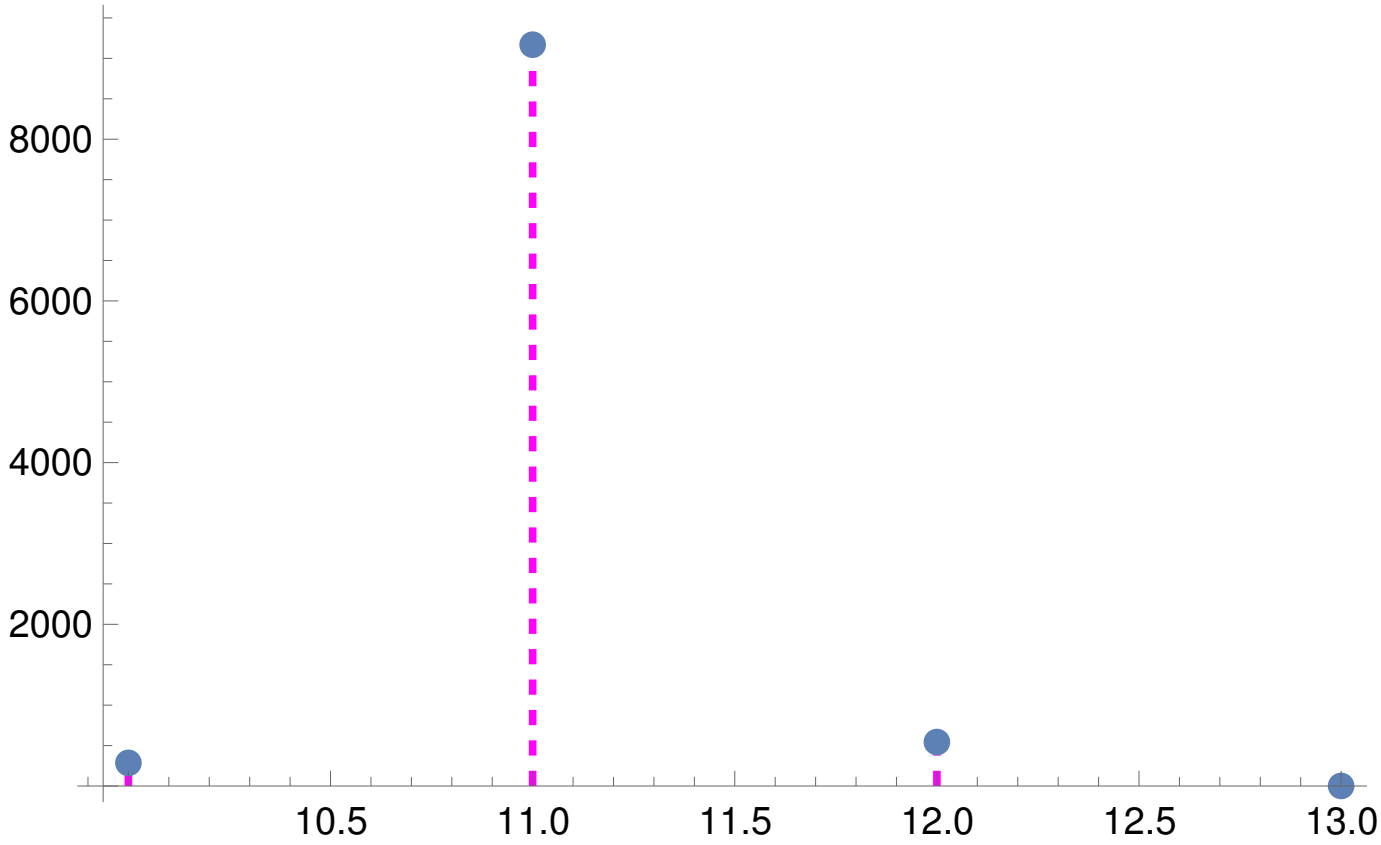
(ii) If $p = 1/n$, then $G_{n,p}$ has a giant component with probability 1 for $n \gg 0$

Giant component:



- Better ($p < \frac{1-\epsilon}{n}$) $\Rightarrow$ a lot of tiny components, ($p = 1/n$) $\Rightarrow$ giant component, ($p > \frac{1+\epsilon}{n}$) $\Rightarrow$ connected
- Random graph theory answers similar questions!

Hard but easy



- The real strength of random graph theory appears for properties that are difficult to compute, e.g. getting the clique number is NP hard
- Above Clique number cl of 10000 $G_{50,1/2}$
- Indeed, the clique number satisfies $cl(G_{n,p}) \approx 2\log_{1/p}(n)$

Thank you for your attention!

I hope that was of some help.