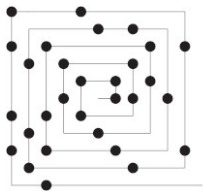


**What is...random graph theory?**

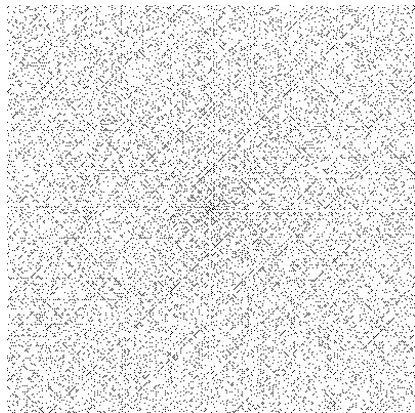
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Or: Subfields of mathematics 5

## Primes are kind of random...



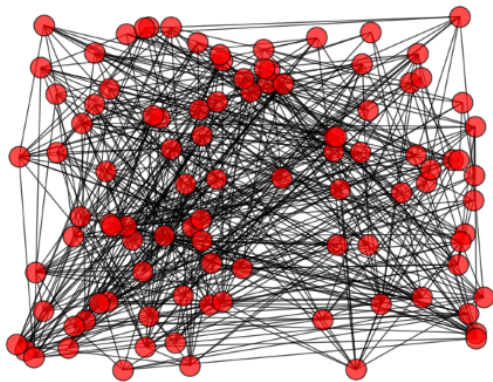
101	100	99	98	97	96	95	94	93	92	91
102	65	64	63	62	61	60	59	58	57	90
103	66	57	56	35	34	33	32	31	56	88
104	67	38	17	16	15	14	13	30	55	88
105	68	39	18	5	4	3	12	29	54	87
106	69	40	19	6	1	2	11	28	33	86
107	70	41	20	7	8	9	10	27	52	85
108	71	42	21	22	23	24	25	26	51	84
109	72	43	44	45	46	47	48	49	50	83
110	73	74	75	76	77	78	79	80	81	82
111	112	113	114	115	116	117	118	119	120	121



- ▶ Prime numbers appear essentially randomly
- ▶ Zooming out, they mostly look like noise
- ▶ However, also many patterns can be observed

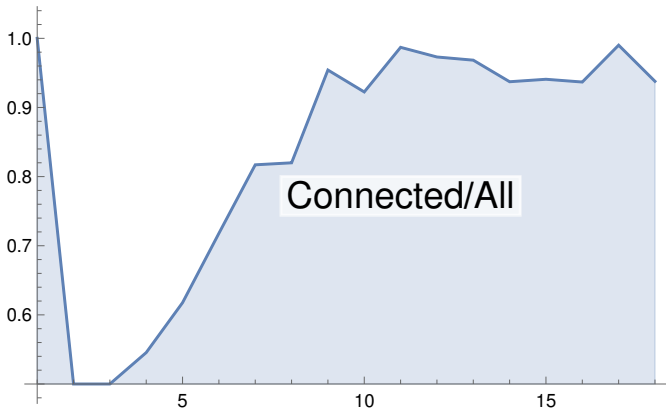
## Graphs are kind of random...

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- 
- ▶ Random graphs = choose edges randomly = “average graphs”
  - ▶ Zooming out, they mostly look like noise
  - ▶ However, also many patterns can be observed

## Patterns in randomness



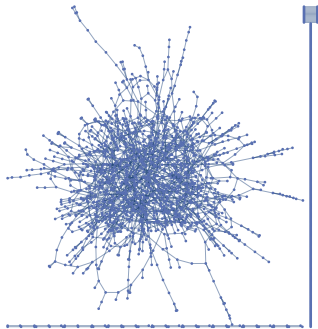
- ▶ **Fact** Most graphs have many edges
- ▶ Random graphs are **almost always** connected (=almost all graphs are connected)
- ▶ **Above**  $\frac{\# \text{ connected graphs}}{\# \text{ all graphs}}$

## Enter, the theorem

$G_{n,p}$  = random graph with  $n$  vertices and probability  $p$  to connect two vertices

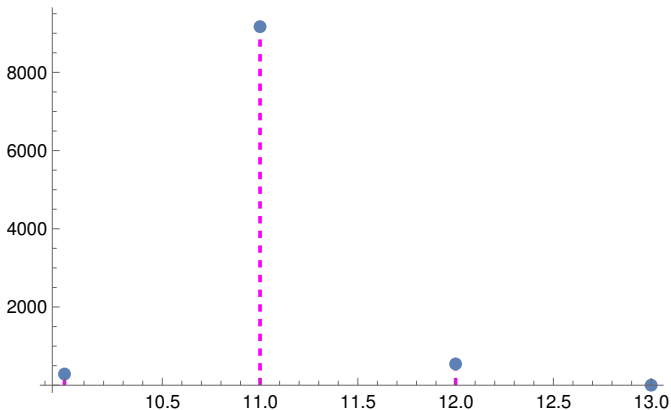
- (i) If  $p$  is constant, then  $G_{n,p}$  is connected with probability 1 for  $n \gg 0$
- (ii) If  $p = 1/n$ , then  $G_{n,p}$  has a giant component with probability 1 for  $n \gg 0$

Giant component:



- ▶ Better ( $p < \frac{1-\epsilon}{n}$ )  $\Rightarrow$  a lot of tiny components, ( $p = 1/n$ )  $\Rightarrow$  giant component, ( $p > \frac{1+\epsilon}{n}$ )  $\Rightarrow$  connected
- ▶ Random graph theory answers similar questions!

## Hard but easy



- ▶ The real strength of random graph theory appears for properties that are difficult to compute, e.g. getting the clique number is NP hard
- ▶ Above Clique number  $cl$  of 10000  $G_{50,1/2}$
- ▶ Indeed, the clique number satisfies  $cl(G_{n,p}) \approx 2 \log_{1/p}(n)$

**Thank you for your attention!**

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I hope that was of some help.