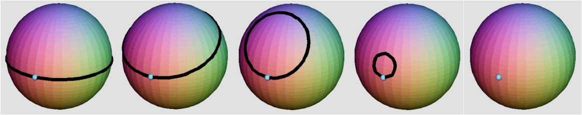
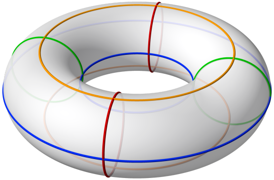
What is...computational topology?

Or: Subfields of mathematics 6

Sphere recognition in 2d





- 2 dim manifolds = surfaces
- Classifying surfaces (up to homeomorphism) has a nice answer
- Observation The only closed sc surface is a sphere = soccer ball
- Simply-connected (sc) = every curve can be shrunk to a point

Sphere recognition in 3d

CINQUIÈME COMPLÉMENT À L'ANALYSIS SITUS.

Par M. H. Poincaré, à Paris.

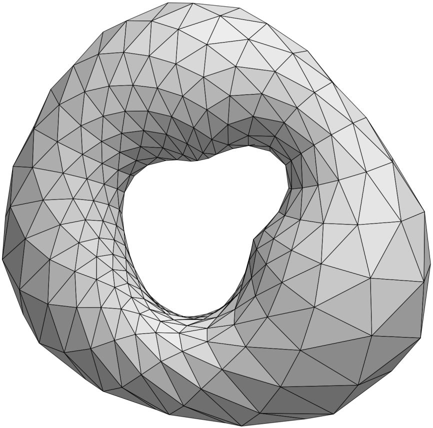
Adunanza del 22 novembre 1903.

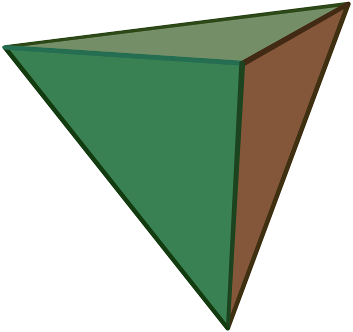
Il resterait une question à traiter :

Est-il possible que le groupe fondamental de V se réduise à la substitution identique, et que pourtant V ne soit pas simplement connexe?

- Closed 3 dim manifolds need four-space to be realized, so are hard to imagine
- Poincaré ∼1904: classification in 3d is difficult, but maybe:
- Question The only closed sc 3 dim manifold is a sphere? True, but difficult

Sphere recognition computationally

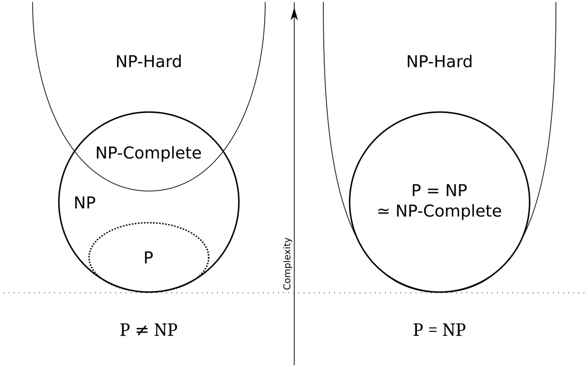




- Rubinstein and Thompson's 3-sphere recognition algorithm (RT algo) = an algorithm to decide whether a 3 dim manifold is a sphere
- Input A triangulated 3 dim manifold
- This means we have a bunch of tetrahedrons

Enter, the theorem

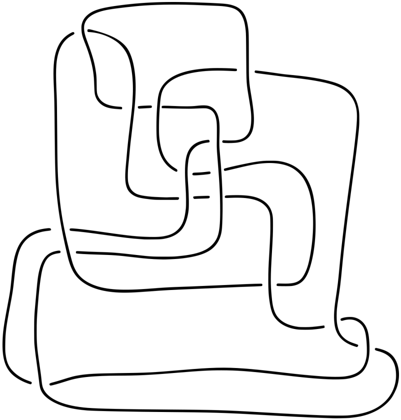
'3-sphere recognition' lies in NP



The RT algo has exponential run-time in the number of tetrahedrons

- '3-sphere recognition' is also in coNP (provided that the generalized Riemann hypothesis holds) Comparison 'Integer factorization' is also in NP and coNP
- Computational topology answers similar questions!

More questions of computational topology



- Theorem 'Unknot recognition' is in NP and coNP
- One looks for ways to efficiently compute homology, knot polynomials, ...
- Programs SnapPea, Regina, ...

Thank you for your attention!

I hope that was of some help.