

What is...analytic combinatorics?

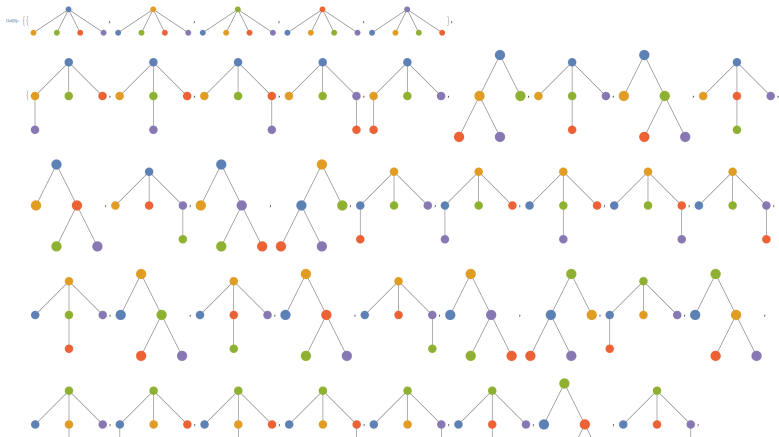
Or: Subfields of mathematics 7

Combinatorics = counting



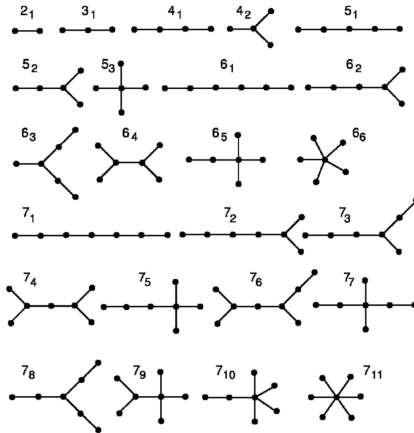
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- ▶ (Enumerative) combinatorics = how many objects of type XYZ are there
 - ▶ Example The number of different possible orderings of a deck of n cards is $n!$
 - ▶ Counting takes many forms: closed formulas, recursions, ...

Good example: counting colored trees



- ▶ The number of colored trees on $n + 1$ vertices is $(n + 1)^{n-1}$
- ▶ Above things are **shifted** and usually people write n^{n-2}
- ▶ Many proofs are known and they are (brilliant but also) **quite easy**

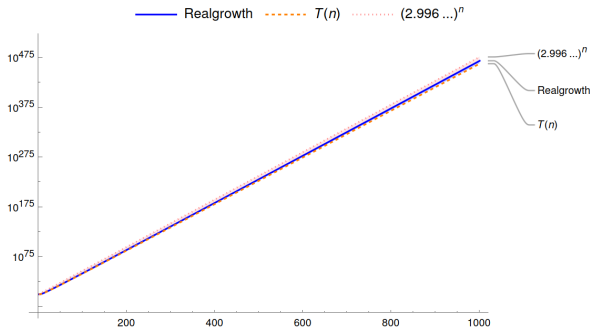
Bad example: counting trees



- ▶ The number of trees on n vertices is ???
- ▶ Turns out that counting trees is very difficult
- ▶ Other non-counting approaches are needed

Enter, the theorem

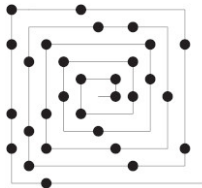
The number of trees $T(n)$ with n vertices satisfies $T(n) \sim \beta \cdot n^{-5/2} \cdot \lambda^n$



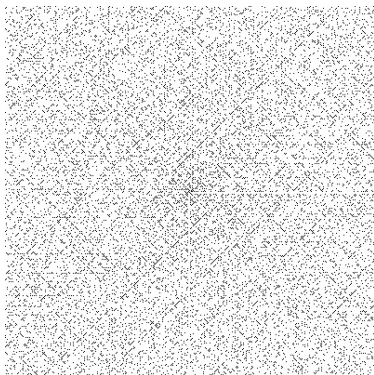
for $\lambda \approx 2.996, \beta \approx 0.535$

- ▶ \sim means asymptotically equal, i.e. the ratio gets to 1
- ▶ This is an example of 'not counting'
- ▶ Analytic combinatorics answers similar questions!

Patterns in randomness



101	100	99	98	97	96	95	94	93	92	91
102	85	64	63	62	61	60	59	58	57	90
103	66	35	36	35	34	33	32	31	56	89
104	87	38	17	16	15	14	13	30	55	88
105	68	39	18	5	4	3	12	28	54	87
106	69	40	19	6	1	2	11	28	53	86
107	70	41	20	7	8	9	10	27	52	85
108	71	42	21	22	23	24	25	26	51	84
109	72	43	44	45	46	47	48	49	50	83
110	73	74	75	76	77	78	79	80	81	82
111	112	113	114	115	116	117	118	119	120	121



- ▶ Prime numbers appear essentially randomly, they mostly look like noise
- ▶ However, also many patterns can be observed
- ▶ Analytic combinatorics takes a similar approach as analytic number theory; just for numbers coming up in combinatorics

Thank you for your attention!

I hope that was of some help.