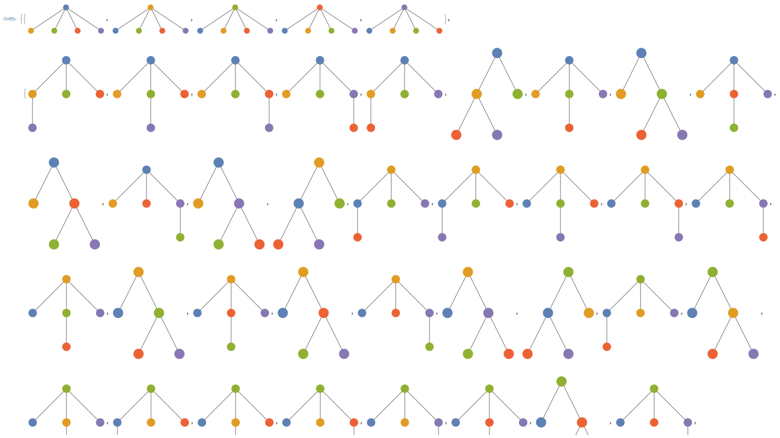
# What is...analytic combinatorics?

Or: Subfields of mathematics 7

# Combinatorics = counting

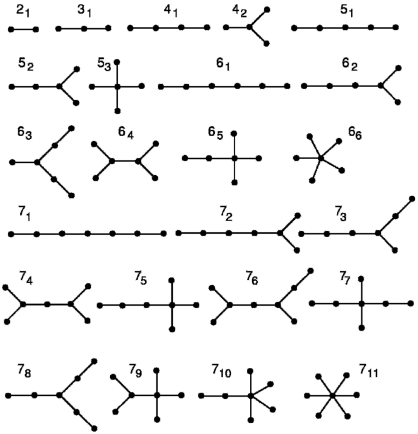
- (Enumerative) combinatorics = how many objects of type XYZ are there
- Example The number of different possible orderings of a deck of $n$ cards is $n!$
- Counting takes many forms: closed formulas, recursions, ...

# Good example: counting colored trees




- The number of colored trees on $n+1$ vertices is $(n+1)^{n-1}$
- Above things are shifted and usually people write $n^{n-2}$
- Many proofs are known and they are (brilliant but also) quite easy

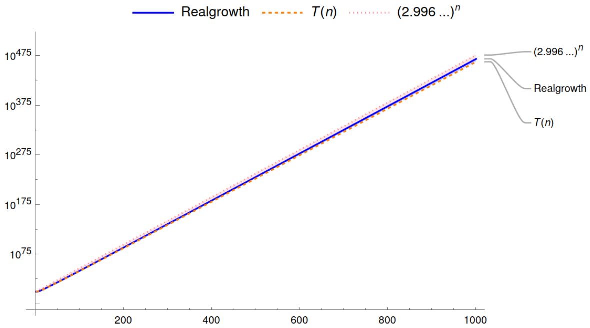
## Bad example: counting trees




- The number of trees on $n$ vertices is ???
- Turns out that counting trees is very difficult
- Other non-counting approaches are needed

# Enter, the theorem

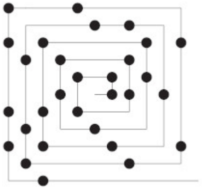
The number of trees $T(n)$ with $n$ vertices satisfies $T(n) \sim \beta \cdot n^{-5/2} \cdot \lambda^n$

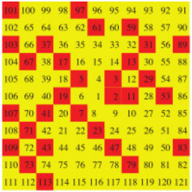


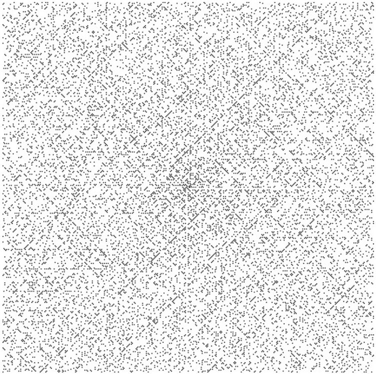

for $\lambda \approx 2.996, \beta \approx 0.535$

- $\sim$ means asymptotically equal, i.e. the ratio gets to 1
- This is an example of 'not counting'
- Analytic combinatorics answers similar questions!

# Patterns in randomness





- Prime numbers appear essentially randomly, they mostly look like noise
- However, also many patterns can be observed
- Analytic combinatorics takes a similar approach as analytic number theory; just for numbers coming up in combinatorics

**Thank you for your attention!**

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I hope that was of some help.