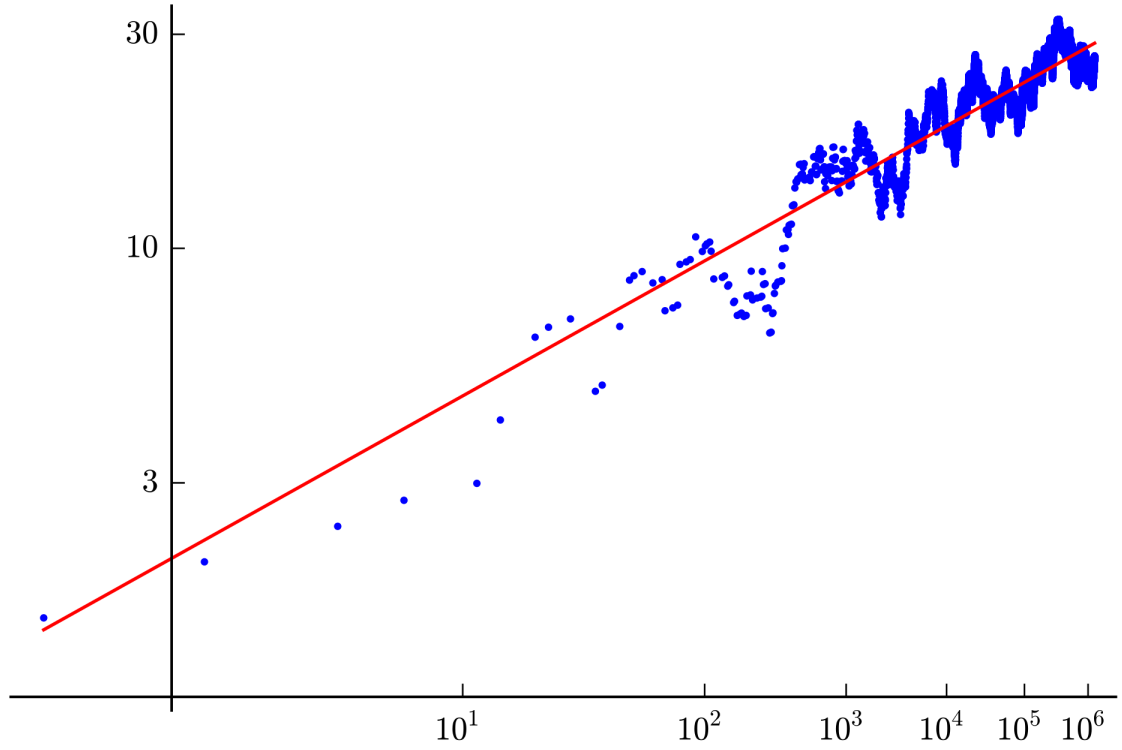
What is...experimental mathematics?

Or: Subfields of mathematics 8

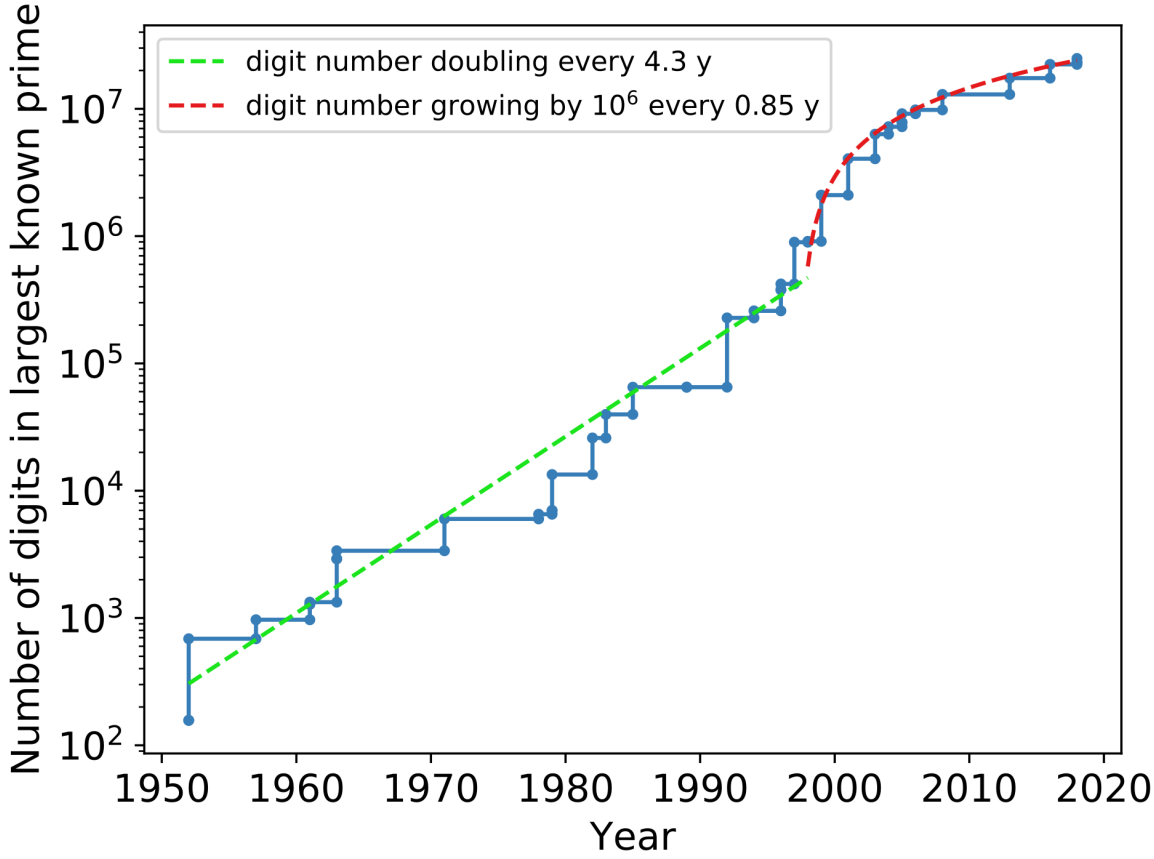
Mathematicians do experiments, too!

The Birch–Swinnerton-Dyer conjecture was discovered using computer experiments



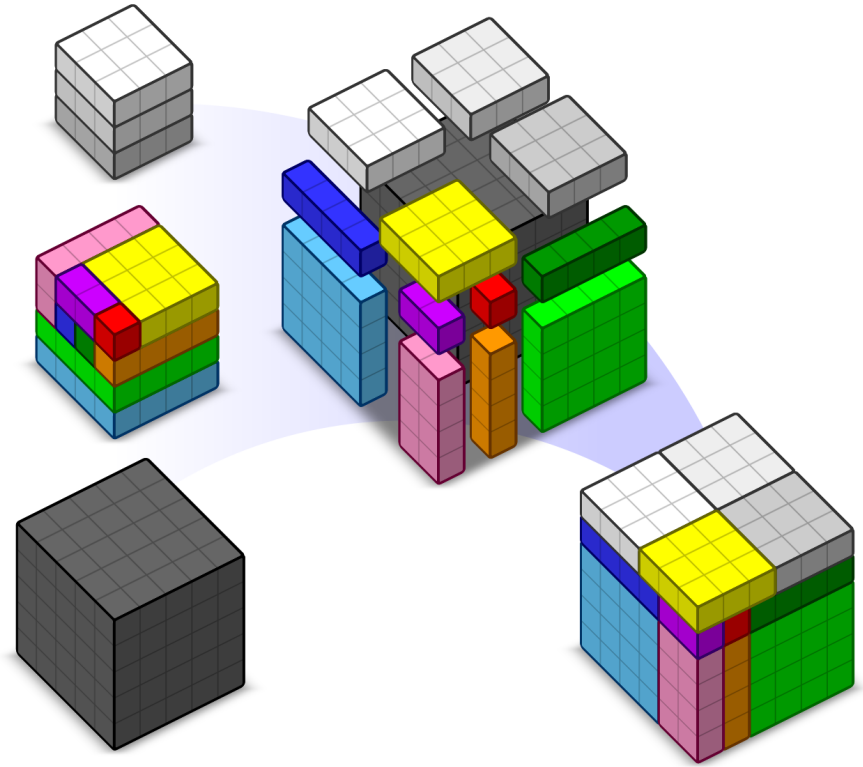
- Experiments are everywhere in mathematics – just in disguise
- Paul Halmos quote Mathematics is not a deductive science—that's a cliché. When you try to prove a theorem, you don't just list the hypotheses, and then start to reason. What you do is trial and error, experimentation, guesswork. You want to find out what the facts are, and what you do is in that respect similar to what a laboratory technician does

Searching for examples



- **Mersenne prime** A prime of the form $2^n - 1$
- **Problem** Its difficult to decide whether $2^n - 1$ is prime for fixed n
- **Experiment** Use a computer to find 'large' Mersenne primes

Searching for counterexamples



$$3^3 + 4^3 + 5^3 = 6^3$$

- Euler's sum of powers conjecture $a_1^k + \ldots + a_n^k = b^k$ implies $n \geq k$
- Problem This is a version of Fermat's last theorem, so expected to be difficult
- Experiment Use a computer to find counterexamples such as $27^5 + 84^5 + 110^5 + 133^5 = 144^5$

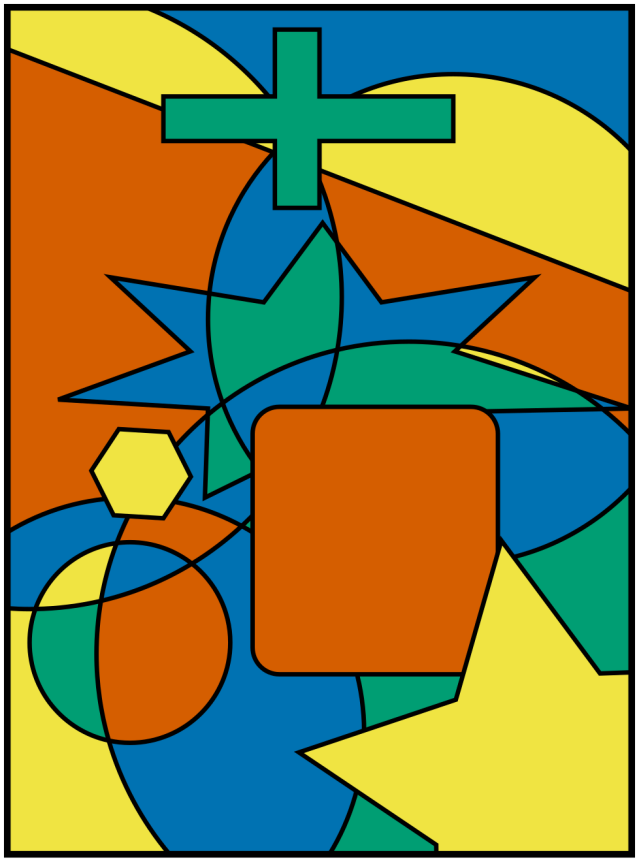
Enter, the theorem

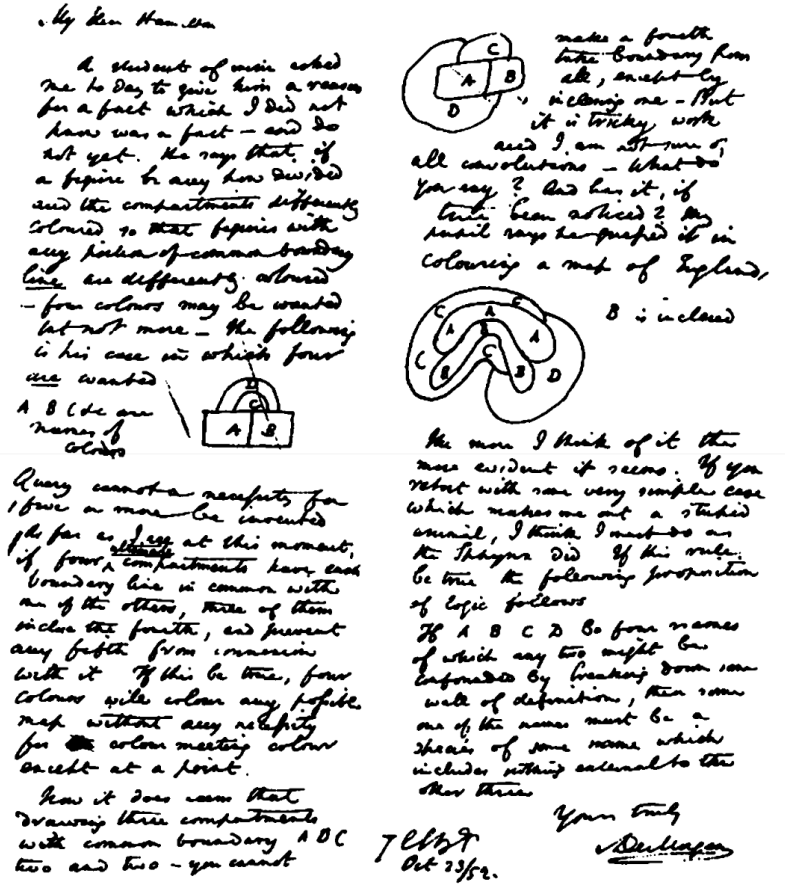
Theorem without words

$$\pi = \sum_{k=0}^{\infty} \left[\frac{1}{16^k} \left(\frac{4}{8k+1} - \frac{2}{8k+4} - \frac{1}{8k+5} - \frac{1}{8k+6} \right) \right]$$

- The formula was discovered using computer experiments
- Honorable mentions Finding patterns (e.g. using visual methods), symbolic validation (e.g. solutions to a special case) etc.
- Experimental mathematics answers similar questions!

Also on the list, but that is a bit strange





- **Four color theorem** Any map can be colored with at most four colors
- **Problem** Every known proof contains too many cases to check by hand
- **Experiment** Use a computer to do the case-by-case check

Thank you for your attention!

I hope that was of some help.