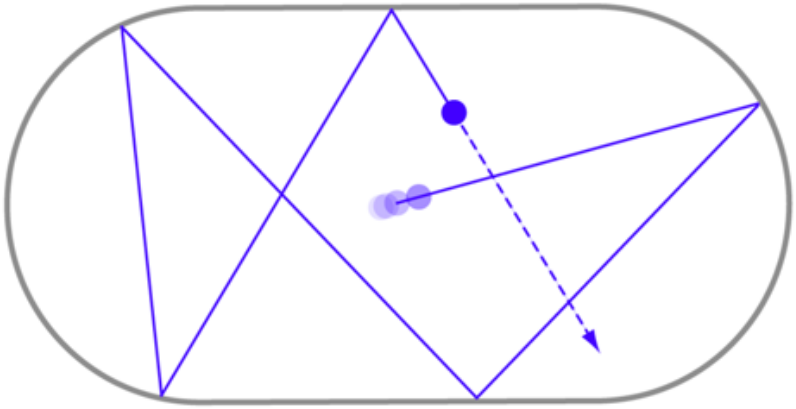
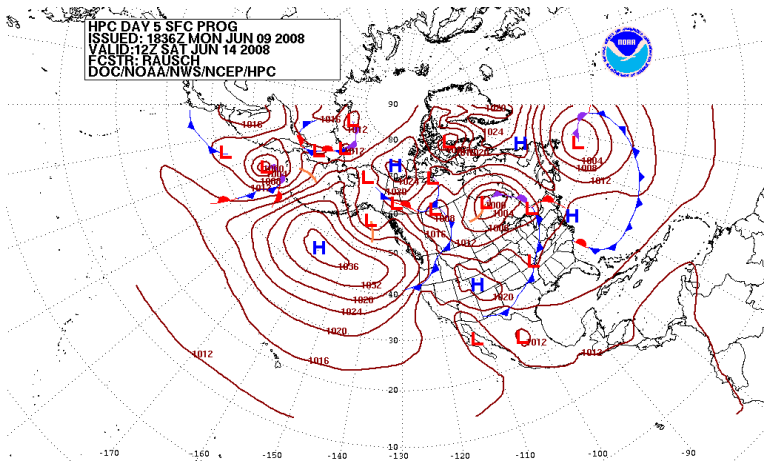
What is...symbolic dynamics?

Or: Subfields of mathematics 9

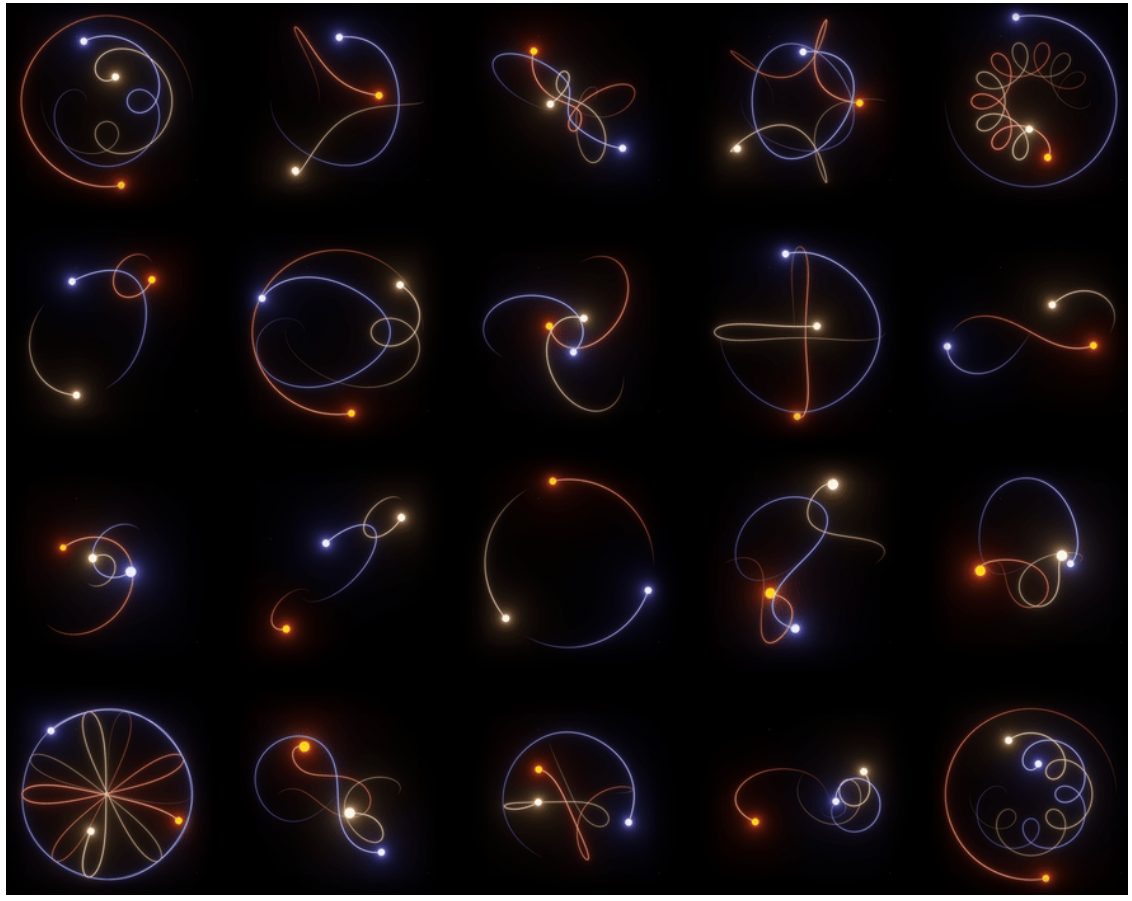
Dynamics





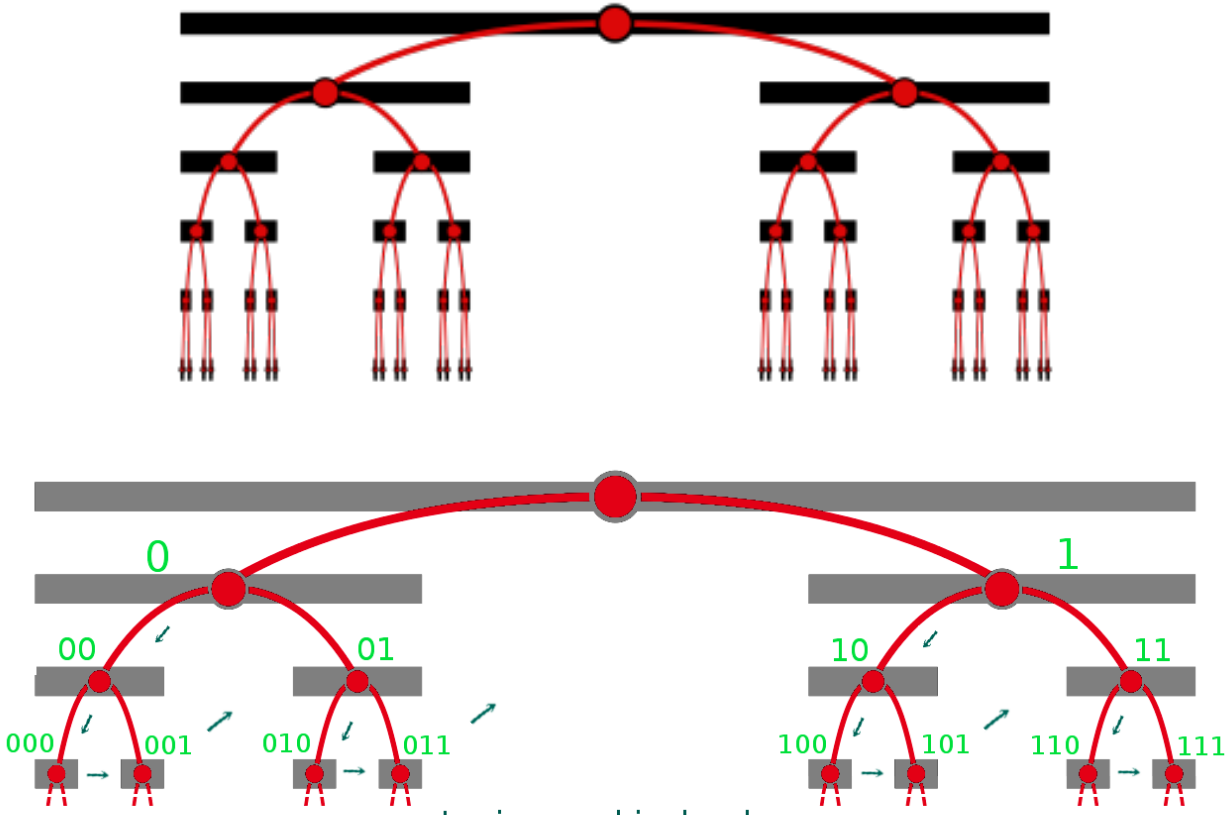
- Dynamical system = particle or particles whose state varies over time
- Example Billiards models, weather etc.
- Time can be measured continuously (say $\mathbb{R}$) or discretely (say $\mathbb{N}$)

A continuous example



- Three body problem = calculate the trajectories of three point masses
- Problem The three-body problem has no general closed-form solution
- This was one of the earliest examples of a chaotic system

Make it discrete

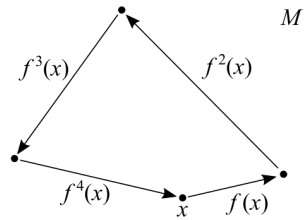


- Symbolic dynamics makes the time discrete
- This results in finite things one can study instead of continuous systems
- Example Instead of the Cantor set one can study binary sequences

Enter, the theorem

Three implies all

period 4:



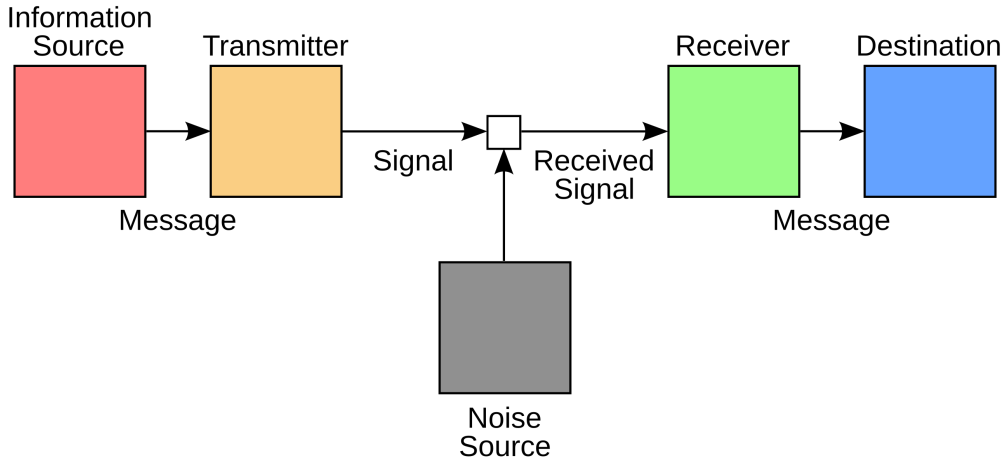
$3 \leq_S 5 \leq_S \ldots$:

$$\begin{array}{ccccccccc} 3 & 5 & 7 & 9 & 11 & \ldots & (2n+1)\cdot 2^0 & \ldots \\ 3\cdot 2 & 5\cdot 2 & 7\cdot 2 & 9\cdot 2 & 11\cdot 2 & \ldots & (2n+1)\cdot 2^1 & \ldots \\ 3\cdot 2^2 & 5\cdot 2^2 & 7\cdot 2^2 & 9\cdot 2^2 & 11\cdot 2^2 & \ldots & (2n+1)\cdot 2^2 & \ldots \\ 3\cdot 2^3 & 5\cdot 2^3 & 7\cdot 2^3 & 9\cdot 2^3 & 11\cdot 2^3 & \ldots & (2n+1)\cdot 2^3 & \ldots \\ & \vdots & & & & & & \\ \ldots & 2^n & \ldots & 2^4 & 2^3 & 2^2 & 2 & 1 \end{array}$$

If a continuous function $f : I \to I$, for some interval $I \subset \mathbb{R}$, has a period of order m, then it has periods of orders $n \geq_S m$

- The order $\leq_S$ is as above
- In particular, period three implies all periods appear
- Symbolic dynamics answers similar questions!

A Mathematical Theory of Communication



The situation can be represented graphically as shown in Figs. 3, 4 and 5. The "states" are the junction

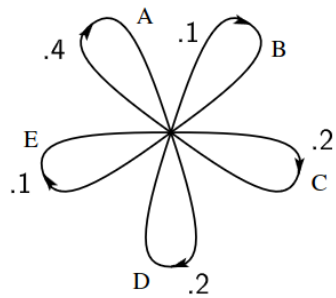


Fig. 3—A graph corresponding to the source in example B.

- The (arguably) most famous application of symbolic dynamics is due to Shannon
- The above paper used symbolic sequences to describe communication
- The above example models a language with 40% A, 10% B, etc.

Thank you for your attention!

I hope that was of some help.