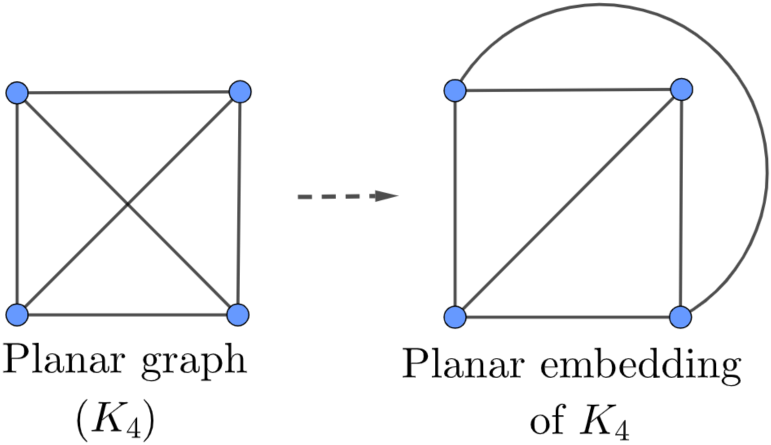
What is...Fáry's theorem?

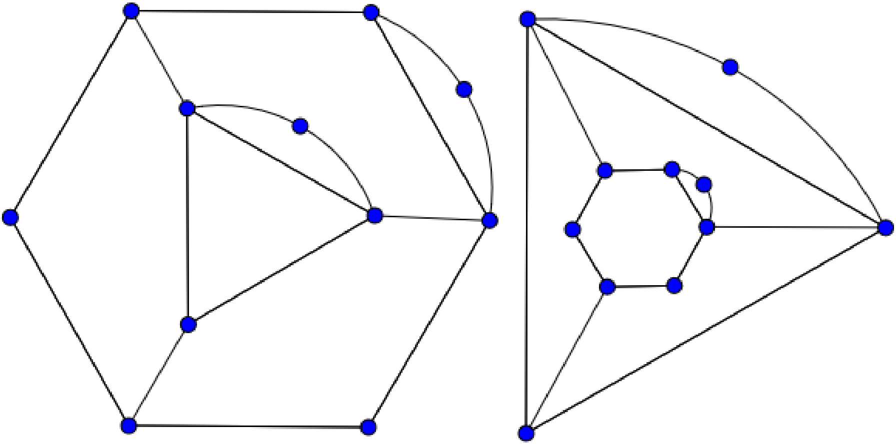
Or: Straight lines everywhere

Graphs in the plane



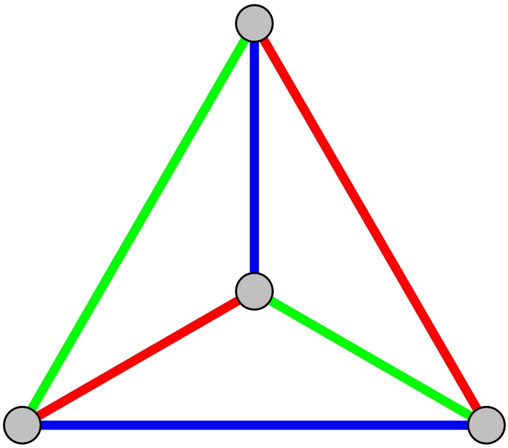
- A planar graph is a graph that can be drawn in the $\mathbb{R}^2$ so that no edges cross
- This gives a planar embedding of the graph
- There are many such planar embeddings

Many planar embeddings



- Planar embeddings can look wildly different
- Question Are there “preferred” planar embeddings?

Use straight lines

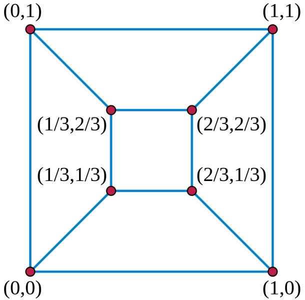


- Above is another planar embedding of the graph from the first slide
- Observe that this embedding only uses straight lines

Enter, the theorem

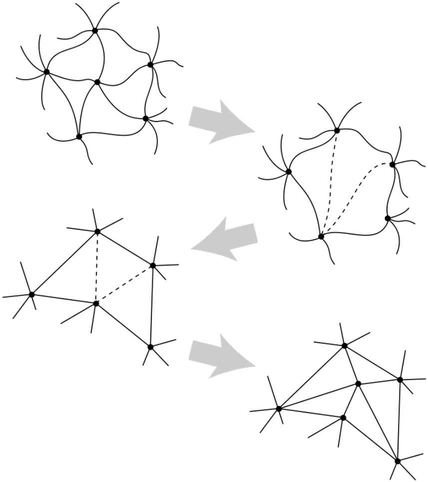
Simple planar graphs can be drawn without crossings so that the edges are straight lines

- There are many related results inspired by Fáry's theorem, *e.g.* Tutte's barycentric embedding:



- Tutte's embedding Every simple 3-vertex-connected planar graph has a Fáry embedding with the outer face being a convex polygon and that each interior vertex is at the average of its neighbors' positions

The proof?



- The proof is a beautiful application of induction on the number of vertices
- Essentially, induction gives the case $|V| < n$ and then fill in the final vertex with straight edges

Thank you for your attention!

I hope that was of some help.