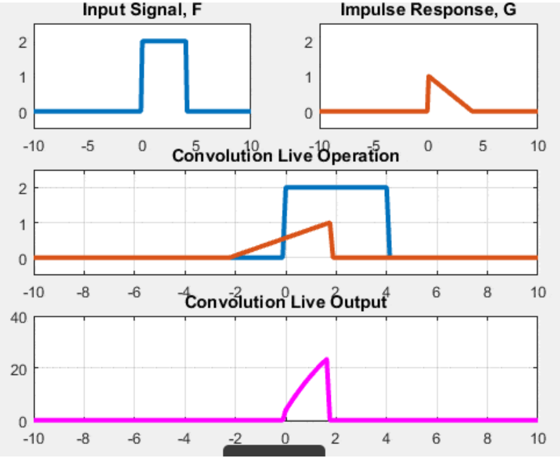
What is...convolution?

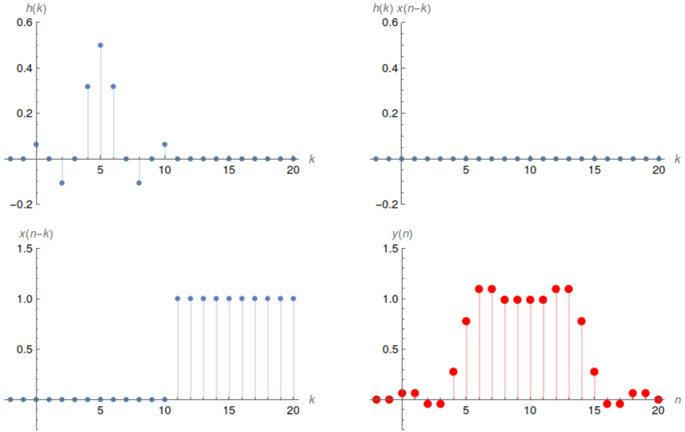
Or: Area, even without area

Classical convolution



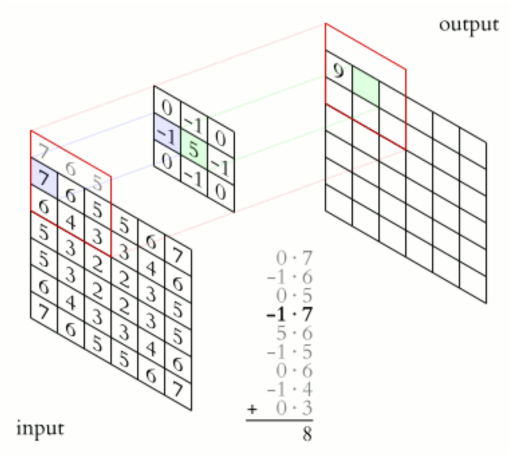
- Take two reasonable functions f, g
- The convolution $f * g$ is the function of the area when sliding g over f
- This originates in Fourier analysis

Discrete convolution



- Take two discrete functions f, g
- The convolution $f * g$ is the function of the sum when sliding g over f
- This originates in Discrete Fourier analysis

Array convolution



- Take two matrices f, g
- The convolution $f * g$ is the function of the sum when sliding g over f
- This originates in signal processing

Enter, the theorem

Convolution is given by some form of the convolution formula

$(f * g)(t) = \int f(x)g(t-x)dx$ $(f * g)(t) = \sum f(x)g(t-x)$

Convolution satisfies:

- Commutativity, associativity, distributivity...
- The set of invertible distributions forms an abelian group under the convolution

- Convolution is very important for fast multiplication (next slide)
- Convolution appears for example in fast multiplication algorithms:

Integer multiplication in time $O(n \log n)$

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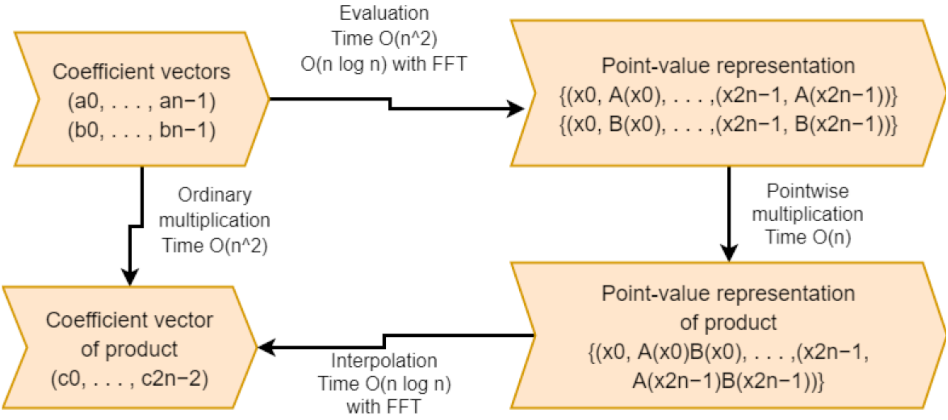
Joint work with Joris van der Hoeven (École Polytechnique, Palaiseau)

Discrete and fast Fourier transform (DFT + FFT)



- Polynomial multiplication cost $O(n^2)$, e.g. $(a, b) \cdot (c, d) = (ac, ad + bc, bd)$
- DFT turns this into pointwise multiplication, which is $O(n)$, via convolution
- FFT computes DFT and DFT^{-1} in $O(n \log n)$

Thank you for your attention!

I hope that was of some help.