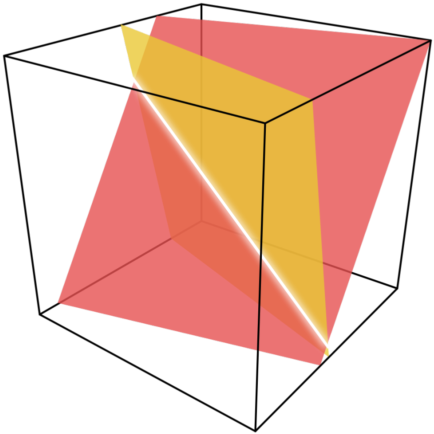
What is...Artin–Wedderburn's theorem?

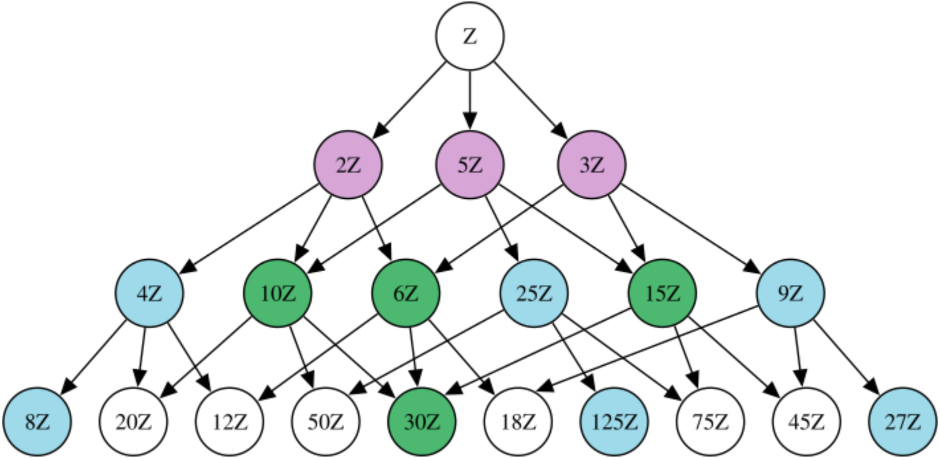
Or: Matrices, of course

Substructure in vector spaces



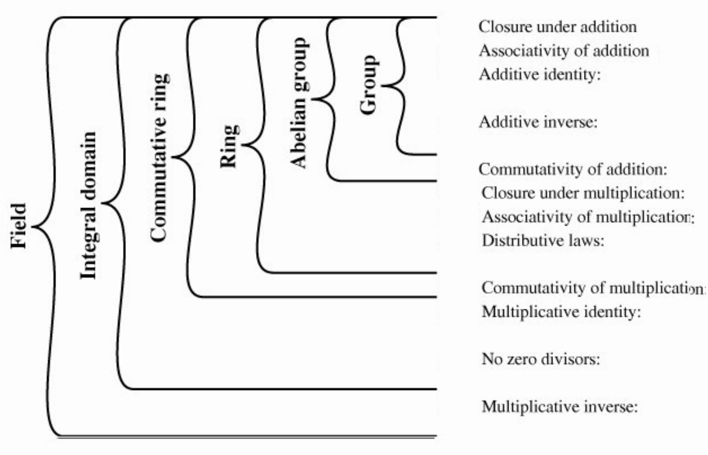
- The “correct” notion of substructure in vector spaces is a linear subspace
- The only vector space without nontrivial substructures is 1d=ground field
- Question What is the analog for rings/algebras?

Substructure in rings



- The "correct" notion of substructure in rings/algebras is a (2-sided) ideal
- The only rings/algebras without nontrivial substructures are called simple
- Question Can we classify them?

Searching for noncommutative fields



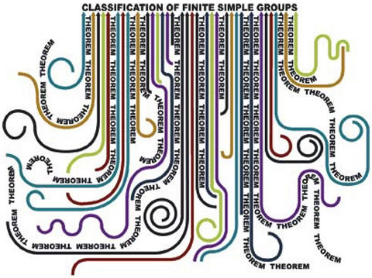
- Fields are the easiest algebraic structures
- A commutative simple ring is a field
- We are thus looking for a "noncommutative analog" of a field

Enter, the theorem

The only simple rings/algebras are matrix rings/algebras (+some finite dimensionality assumption)

$$\begin{array}{c|cccc|} & 1 & 2 & \cdots & n \\ 1 & a_{11} & a_{12} & \ldots & a_{1n} \\ 2 & a_{21} & a_{22} & \ldots & a_{2n} \\ 3 & a_{31} & a_{32} & \ldots & a_{3n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ m & a_{m1} & a_{m2} & \ldots & a_{mn} \end{array}$$

- This is surprisingly easy compared to the classification of other "simple things"
- For example, the classification of finite simple groups is very difficult



The more general theorem

Character Table of S_3

	IDENTITY 1	REFLECTION s_1	ROTATION $s_1 s_2$
Trivial representation	1	1	1
Reflection representation	2	0	-1
Sign representation	1	-1	1

- There is a more general version which is very useful in representation theory
- Theorem Semisimple rings/algebras are direct sums of matrix rings/algebras

Thank you for your attention!

I hope that was of some help.