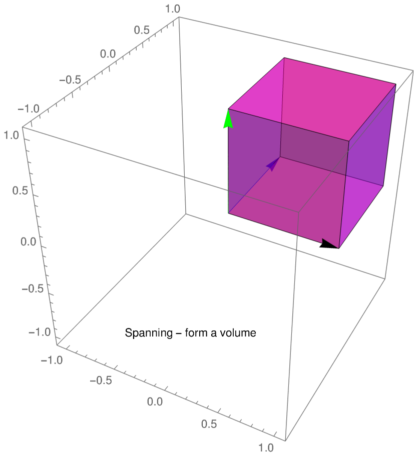
What is...Drozd's theorem?

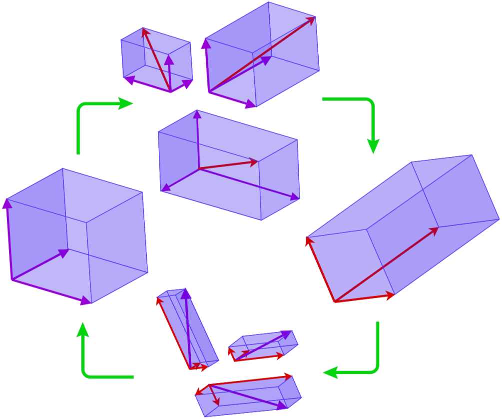
Or: The Jordan normal form doesn't get better...

Let us start with vector spaces



- Task Classify vector spaces up to isomorphism
- Solution The dimensions determines the vector space
- Thus, vector spaces are classified by one discrete parameter

Now: endomorphisms of vector spaces



- A natural equivalence relation on matrices is similarity:

$$(A \sim B) \Leftrightarrow (\exists P : A = P^{-1}BP)$$

Similarity = A and B are the same matrix up to base change

- Question How can we classify similar matrices, say over $\mathbb{C}$?

The Jordan normal form (JNF)

$$
\begin{pmatrix}
\boxed{\begin{matrix}\lambda_1 & 1 & \\ & \lambda_1 & 1 \\ & & \lambda_1\end{matrix}} & & & & \\
& \boxed{\begin{matrix}\lambda_2 & 1 \\ & \lambda_2\end{matrix}} & & & \\
& & \boxed{\lambda_3} & & \\
& & & \ddots & \\
& & & & \boxed{\begin{matrix}\lambda_n & 1 \\ & \lambda_n\end{matrix}}
\end{pmatrix}
$$

- Theorem Two matrices are similar if and only if they have the same JNF
- Thus, similarity is classified by:

finitely many discrete parameters = sizes of Jordan blocks

finitely many continuous parameters = eigenvalues

Enter, the theorem

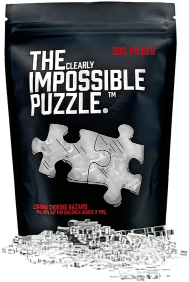
Trichotomy theorem Exactly one of the following holds for A-modules:

(1) The indecomposables are classified by finitely many discrete parameters

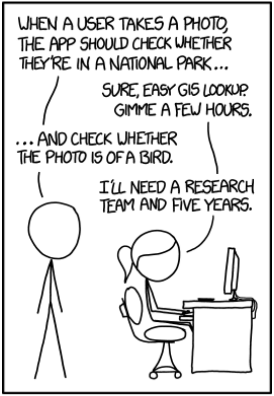
(2) The indecomposables are classified by finitely many discrete and continuous parameters

(3) There is no classification scheme

- $A =$ some fin dim algebra
- Indecomposable = elements = $X \cong Y \oplus Z$ implies Y or Z is zero
- Thus, classification is like for vector spaces, for similarity or impossible



Its a fine line



IN CS, IT CAN BE HARD TO EXPLAIN THE DIFFERENCE BETWEEN THE EASY AND THE VIRTUALLY IMPOSSIBLE.

- Similarity has a nice solution
- Simultaneous similarity $(A, B) \sim (P^{-1}AP, P^{-1}BP)$ is extremely difficult

Thank you for your attention!

I hope that was of some help.