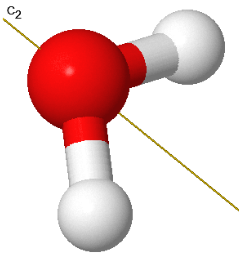
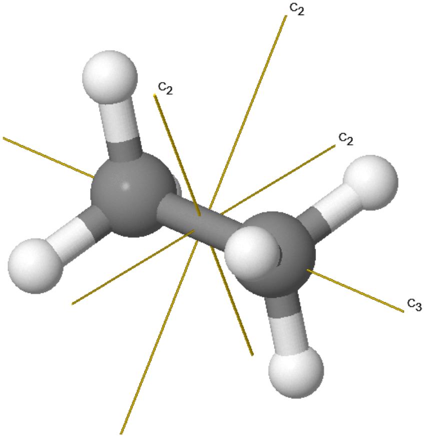
What are...Kronecker coefficients?

Or: Terribly difficult

Symmetry is everywhere

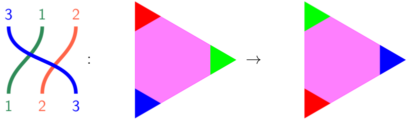


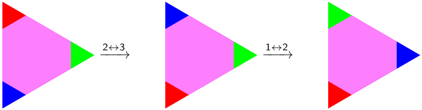


- Symmetry can be mathematically modeled using representations
- The elements of the theory are called simples
- The study of simples corresponds to the study of the basic symmetries

Symmetry type = groups

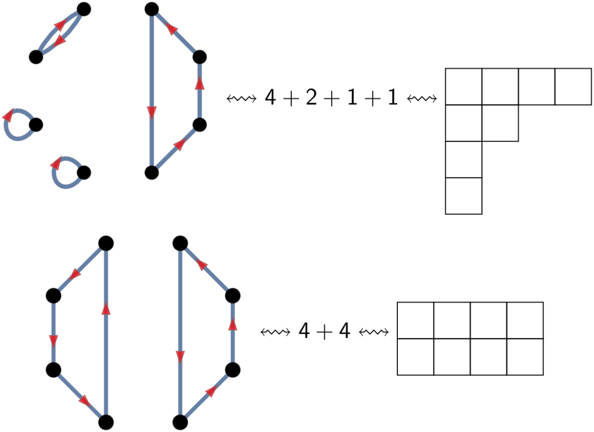
The symmetric group in three letters acts on a triangle via the rule "green=1, red=2, blue=3, and then permute":





- One categorizes symmetries by type which mathematically speaking are group
- Example A symmetric group symmetry is some form of shuffle symmetry
- Example A triangle also has a shuffle-type symmetry

They are actually not that difficult!



- Task Classify all the simple = elements shuffle symmetries
- Frobenius ∼1895 There is a very satisfying answer
- The simple shuffle symmetries V_λ are indexed by partitions λ and much more is known about them (dimensions, characters...)

Enter, the theorem

Regarding products of shuffle symmetries:

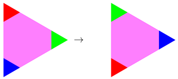
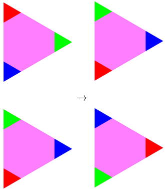
(i) Existence (easy) There exist $g_{\lambda,\mu}^{\kappa} \in \mathbb{N}$ such that

$$V_\lambda \otimes V_\mu \cong \bigoplus_{\kappa} V_\kappa^{\oplus g_{\lambda,\mu}^{\kappa}}$$

(ii) Deciding whether $g_{\lambda,\mu}^{\kappa} \neq 0$ is NP-hard (difficult)

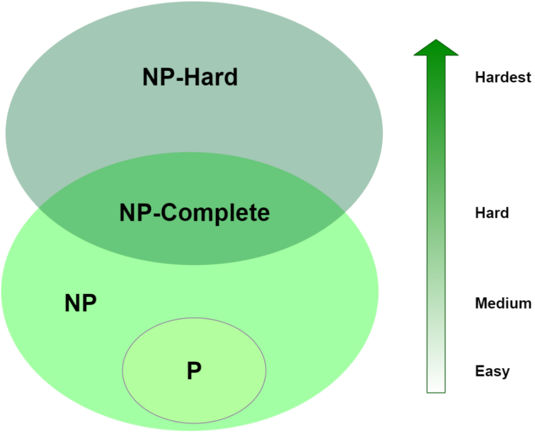
(iii) Computing $g_{\lambda,\mu}^{\kappa}$ is #P-hard (super difficult)

- Note the complexity jump from one factor to two factors

easy:  , difficult: 

- This is just the tip of the iceberg: product are often very difficult

Terrible, but maybe not



- A lot of "difficult" problems are actually easy on large subclasses
- Example The Hamiltonian path problem is very difficult in general but e.g. easy on 4-connected planar graphs
- The same happens for the $g^{\kappa}_{\lambda,\mu}$: they are often easy to compute

Thank you for your attention!

I hope that was of some help.