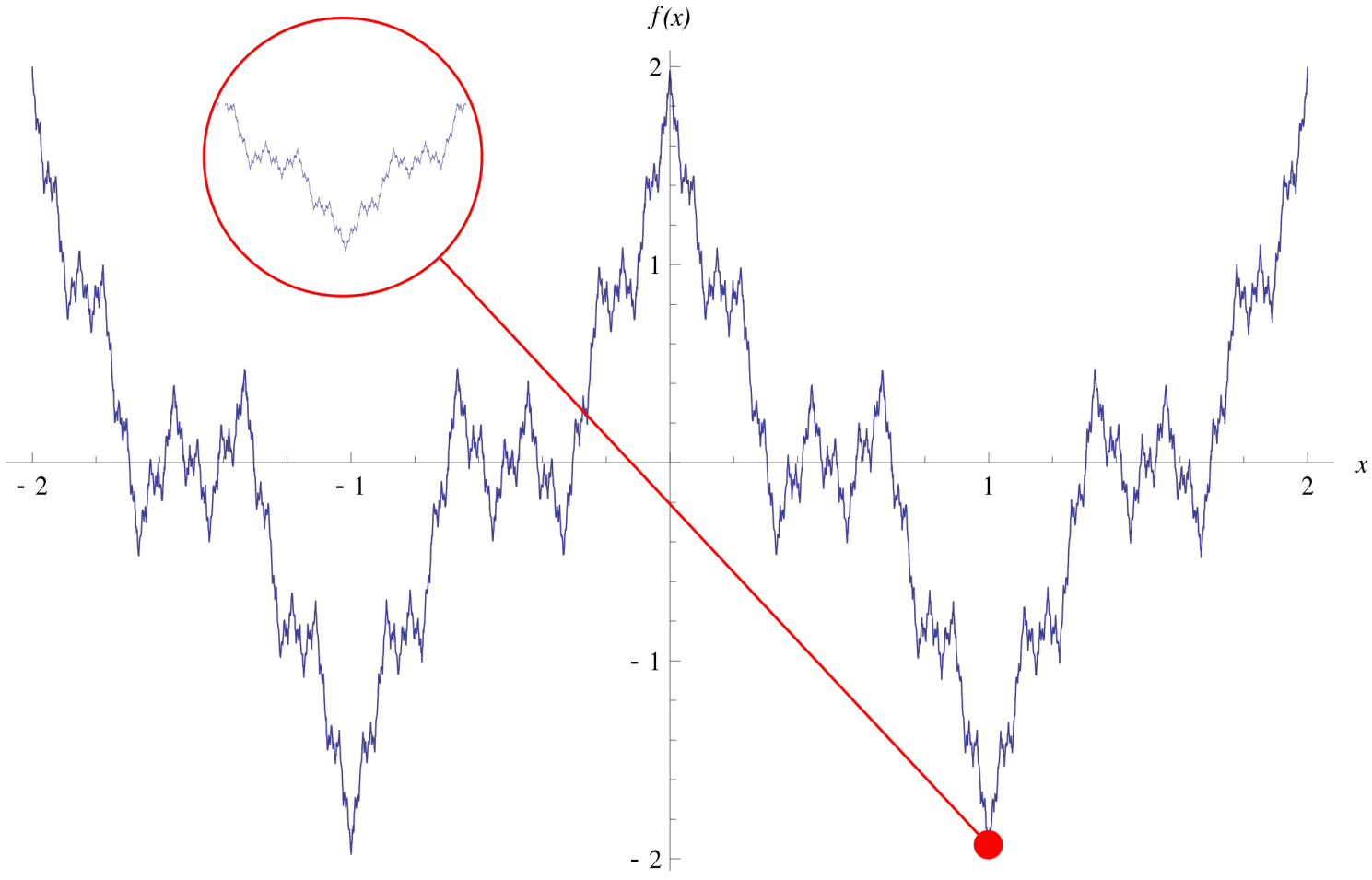
What is...a fractal?

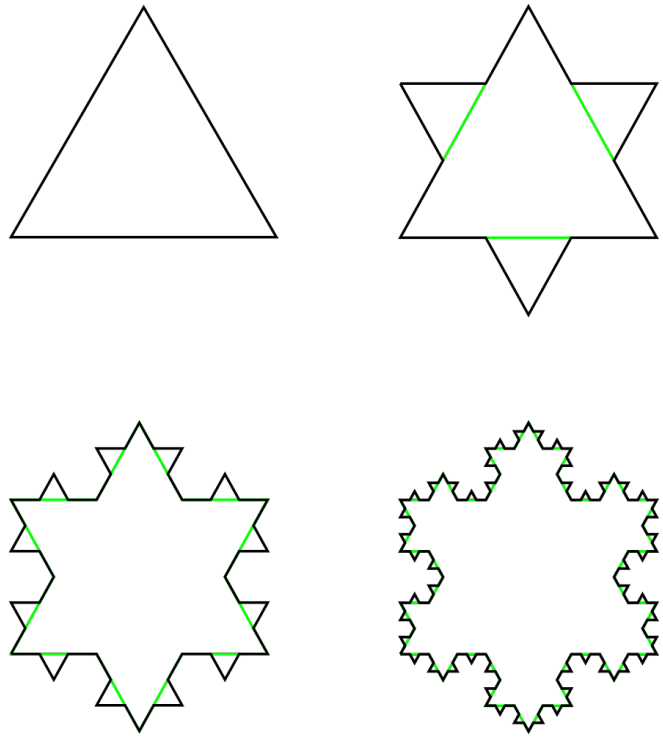
Or: Zooming in

History of fractals - part I



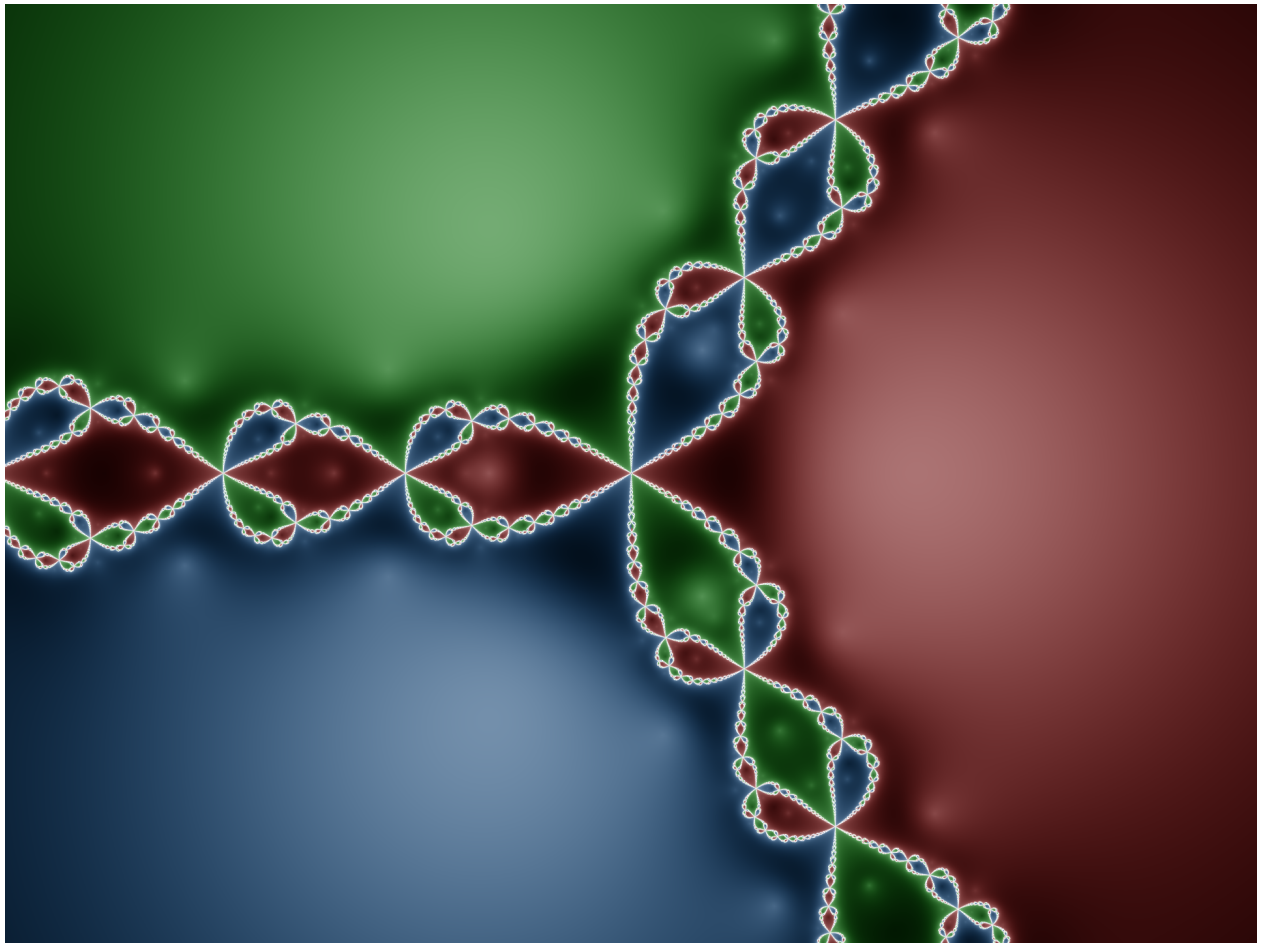
- Fractal = something that is self-similar
- Analysis definition Fractal = nowhere differentiable function
- Above Historically the first example is Weierstrass' curve

History of fractals - part II



- Fractal = something that is self-similar
- Geometry definition Fractal = the limit of some iteration
- Above An early example is Koch's snowflake curve

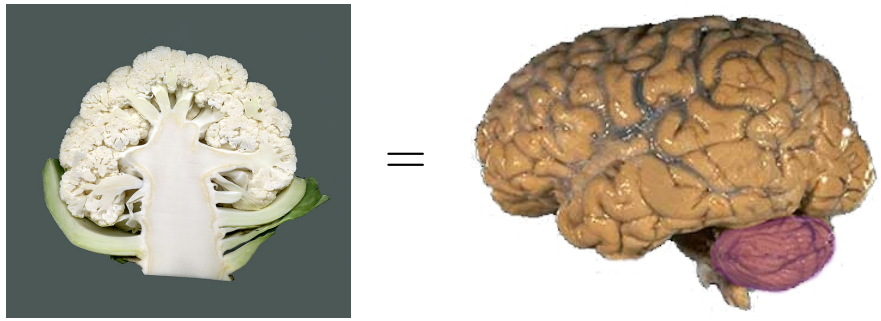
History of fractals - part III



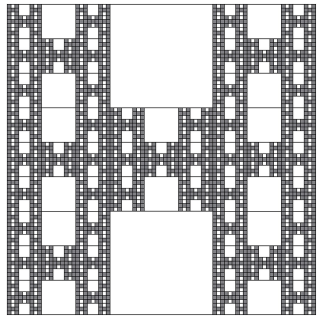
- Fractal = something that is self-similar
- Dynamics definition Fractal = the boarder between order and chaos
- Above An early example is Fatou and Julia sets for Newton's method

Enter, the theorem

Cauliflower has the same dimension as then human brain

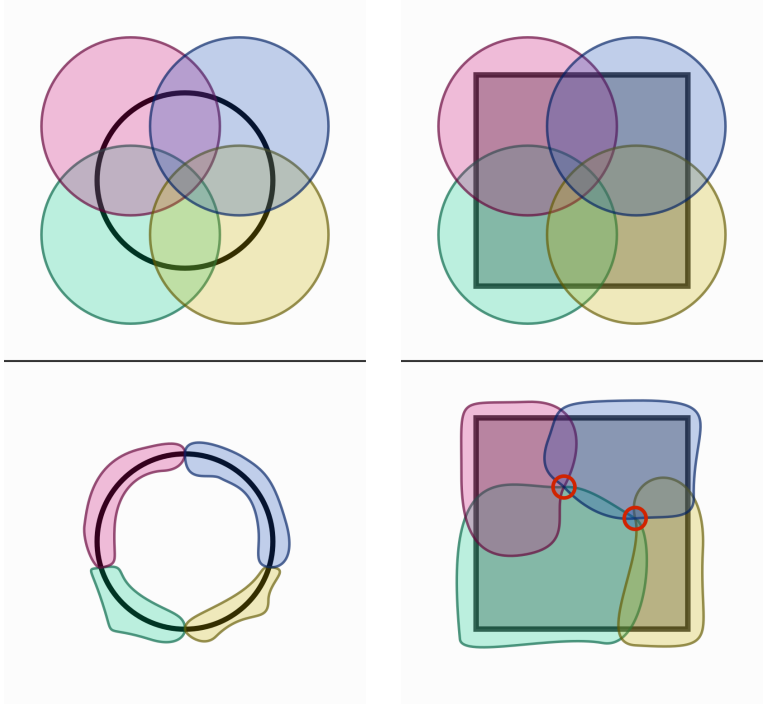


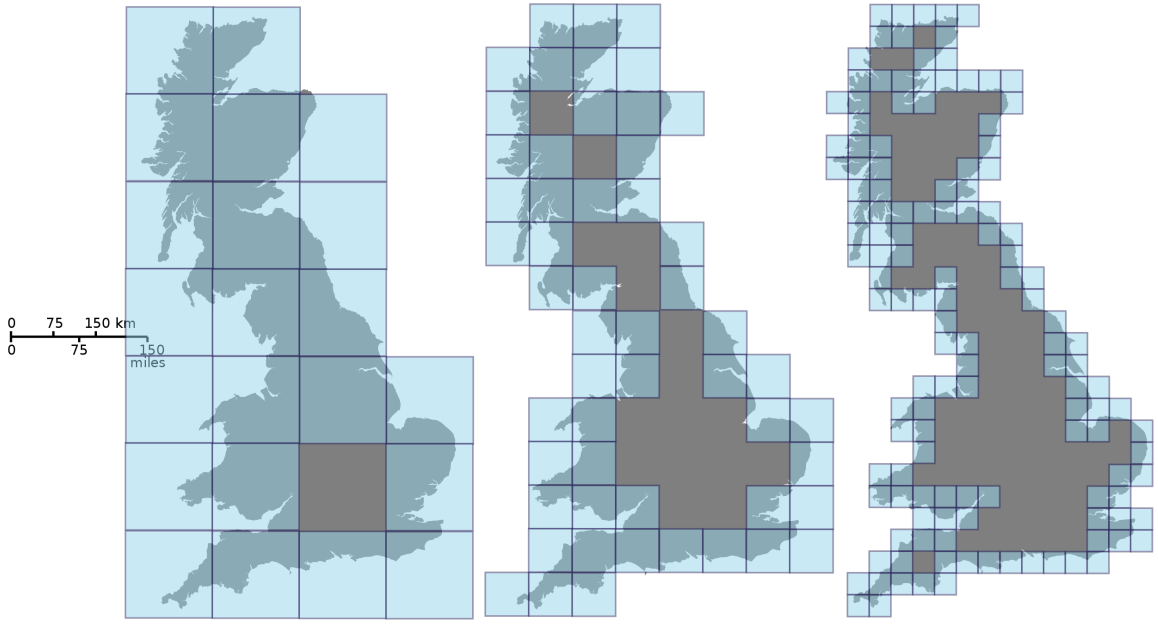
- Dimension here means Hausdorff dimension (very briefly on the next slide)
- Fractals were popularized during the age of computer generated graphics



- "Definition" A fractal is a set for which the Hausdorff dimension strictly exceeds the topological dimension

Enter, Hausdorff





- Topological dimension (left) = minimum n such that every point lies in the intersection of no more than $n+1$ covering sets
- Hausdorff dimension (no where) = annoying to write down but almost the box dimension
- Box dimension (right) = $\lim_{\epsilon \to 0} \frac{\log N(\epsilon)}{\log 1/\epsilon}$ where $N(\epsilon)$ is the number of boxes of side length ϵ for a cover

Thank you for your attention!

I hope that was of some help.